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Portfolio Approaches in Constraint Programming

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Abstract

Recent research has shown that the performance of a single, arbitrarily efficient algorithm can be significantly outperformed by using a *portfolio* of —possibly on-average slower— algorithms. Within the *Constraint Programming* (CP) context, a *portfolio solver* can be seen as a particular constraint solver that exploits the synergy between the constituent solvers of its portfolio for predicting which is (or which are) the best solver(s) to run for solving a new, unseen instance.

In this thesis we examine the benefits of portfolio solvers in CP. Despite portfolio approaches have been extensively studied for Boolean Satisfiability (SAT) problems, in the more general CP field these techniques have been only marginally studied and used. We conducted this work through the investigation, the analysis and the construction of several portfolio approaches for solving both satisfaction and optimization problems. We focused in particular on *sequential* approaches, i.e., single-threaded portfolio solvers always running on the same core.

We started from a first empirical evaluation on portfolio approaches for solving *Constraint Satisfaction Problems* (CSPs), and then we improved on it by introducing new data, solvers, features, algorithms, and tools. Afterwards, we addressed the more general *Constraint Optimization Problems* (COPs) by implementing and testing a number of models for dealing with COP portfolio solvers. Finally, we have come full circle by developing `sunny-cp`: a sequential CP portfolio solver that turned out to be competitive also in the MiniZinc Challenge, the reference competition for CP solvers.

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"Gratus animus est una virtus non solum maxima,
sed etiam mater virtutum omnium reliquarum"[1]
Marcus Tullius Cicero.

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[1] "Gratitude is not only the greatest of virtues, but the parent of all others".

Silvio Peroni, Francesco Poggi, Giuseppe Profiti, Alessandro Rioli, Imane Sefrioui, Francesco Strappaveccia.

Almost all the work described in this thesis has been conducted together with Prof. Maurizio Gabbrielli, the Ph.D. program coordinator as well as my supervisor, and with Dr. Jacopo Mauro.

Let me say that it is not uncommon to hear about Ph.D. students treated as employees, if not servants, by their supervisors. On the contrary, Maurizio is a very good man and has been for me an ideal supervisor: he never put pressure on me, leaving me almost total freedom of research.

Jacopo introduced me to Algorithm Portfolios, and has been the colleague that most helped me during the doctorate course. We had sometimes lively debates, but without his help it would have been very difficult to get the appreciable results we achieved during these years.

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Contents

Chapter 1

Introduction

"Dimidium facti, qui coepit, habet"[1]

Quintus Horatius Flaccus.

The *Constraint Programming* (CP) paradigm is a general and powerful framework that enables to express relations between different entities in form of *constraints* that must be satisfied. The concept of constraint is ubiquitous and not confined to the sciences: constraints appear in every aspect of daily life in the form of requirements, obligations, or prohibitions. For example, logistic problems like task scheduling or resource allocation can be addressed by CP in a natural and elegant way. The CP paradigm, characterized as a sub-area of Artificial Intelligence, is essentially based on two vertical layers:

(i) a *modelling* level, in which a real-life problem is identified, examined, and formalized into a mathematical model by human experts;

(ii) a *solving* level, aimed at resolving as efficiently and comprehensively as possible the model defined in (i) by means of software agents called *constraint solvers*.

[1] "Well begun is half done".

The goal of CP is to model and solve *Constraint Satisfaction Problems* (CSPs) as well as *Constraint Optimization Problems* (COPs) [159]. Solving a CSP means to find a solution that satisfies all the constraints of the problem. A COP is instead a generalized CSP where we are also interested in optimal solutions, i.e., solutions that minimize a cost or maximize a payoff.

The *Algorithm Selection* (AS) problem, for the first time formalized by John R. Rice in 1976 [157], can be roughly reduced to the following question:

Given an input problem P and a set $\mathcal{A} = \{A_1, A_2, \ldots, A_n\}$ of algorithms, which is the "best" algorithm $A_i \in \mathcal{A}$ for solving problem P?

The underlying idea behind AS is very general and implicitly used in practical everyday human problems. For instance, let us suppose the problem P is "I have to go from place x to place y" and the algorithm space is $\mathcal{A} = \{$"Go from x to y by M" : $M \in \{$car, train, plane$\}\}$. The selection necessary depends on the input problem parameters x and y: e.g., if we have to move from Bologna to Melbourne, the choice is obvious and strongly dominated by the the distance between x and y. Less clear would be instead the choice if $x =$ "Bologna" and $y =$ "Roma". Here other problem *features* must be taken into account: e.g., the possible heavy traffic in the *raccordo anulare* of Roma[2], or the remote possibility of train strikes or delays. Moreover, note that the definition of "best" algorithm is not self-contained and generally bound to a *performance metric*: the best path from x is to y is the shortest, the fastest, the cheaper, or what?

According to [118] definition, *Algorithm Portfolios* [103] can be seen as particular instances of the more general AS problem in which the algorithm selection is performed case-by-case instead that in advance. In this case, a *portfolio* of algorithms corresponds to the algorithm space $\mathcal{A}$. The concept of portfolio comes from Economics, and refers to a collection of financial assets used for maximizing the expected return and minimizing the overall risk of having just a single stock. Within the Artificial Intelligence field, algorithm portfolios have been deeply investigated

[2] `http://en.wikipedia.org/wiki/Grande_Raccordo_Anulare`

by Gomes and Selman [83, 82, 84]. Within the sub-area of CP the algorithm space is constituted by a portfolio $\{s_1, s_2, \ldots, s_m\}$ of different constraint solvers s_i. We can thus define a *portfolio solver* as a particular constraint solver that exploits the synergy between the constituent solvers of its portfolio. When a new unseen problem p comes, the portfolio solver tries to predict which is (or which are) the best constituent solver(s) $s_1, s_2, \ldots, s_k$ ($k \leq m$) for solving p and then runs such solver(s) on p.

The goal of this thesis is to examine the benefits of portfolio approaches in Constraint Programming. From this perspective the state of the art of portfolio solvers is still a raw fruit if compared, e.g., to the SAT field where a number of effective portfolio approaches have been developed and tested. We constructed, analysed and evaluated several portfolio approaches for solving generic CP problems (i.e., both CSPs and COPs). We started with satisfaction problems, then we moved to optimization ones and finally we have come full circle by developing `sunny-cp`: a portfolio solver for solving generic CP problems that turned out to be competitive also in the *MiniZinc Challenge* [176, 177], the reference competition for CP solvers. A more detailed outline of the contents and the contributions of this thesis is reported in the next section.

1.1 Outline

This thesis is conceptually organized in three main parts. The first part (Chapters 2 and 3) contains the background notions about CP and portfolio solvers. The second one (Chapters 4 and 5) presents detailed and extensive evaluations on portfolio approaches applied to CSPs and COPs respectively. The last part describes the `sunny-cp` tool (Chapter 6) and concludes the thesis by reporting also the ongoing and future works (Chapter 7). Hereinafter we show an outline of these chapters.

Chapter 2 provides an overview of Constraint Programming: the main domains, solving techniques, and approaches for solving both CSPs and COPs.

Chapter 3 introduces the main ingredients that characterize a portfolio solver,

namely: the dataset of instances used to make (and test) predictions, the constituent solvers of the portfolio, the features used to characterize each problem, and the techniques used to properly select and execute the constituent solvers.

Chapter 4 presents an extensive investigation on CSP portfolio solvers. We started from an initial empirical evaluation, where we compared by means of simulations some state-of-the-art portfolio solvers against some simpler approaches built on top of some Machine Learning classifiers. Then we report a more extensive evaluation in which, in particular, we introduce SUNNY: a new algorithm portfolio for constraint solving.

Chapter 5 shifts the focus on optimization solvers. We first formalized a suitable model for adapting the "classical" satisfaction-based portfolios to address COPs, providing also some metrics to measure the solver performance. Then, we simulated and evaluated the performances of different COP portfolio solvers. After this work, we take a step forward by assessing the benefits of sequential time splitting and bounds communication between different COP solvers.

Chapter 6 describes `sunny-cp`: a sequential CP portfolio solver that can be run just like a regular single constraint solver. We show the overall architecture of `sunny-cp` and we assess its performance according to the good results it achieved in the MiniZinc Challenge 2014.

Chapter 7 concludes the thesis by providing some insights and final remarks on the work we have done, we are doing and we are planning to do.

Summarizing, the contributions of the thesis are the followings:

- we built solid baselines by performing several empirical evaluations of different portfolio approaches for solving both CSPs and COPs;
- we provided —by means of proper tools— the full support of MiniZinc, nowadays a de-facto standard for modelling and solving CP problems, while still retaining the compatibility with XCSP format;
- we developed SUNNY, a lazy portfolio approach for constraint solving. SUNNY is a simple and flexible algorithm portfolio that, without building an ex-

plicit prediction model, predicts a schedule of the constituent solvers for solving a given problem.

- we took a step forward towards the definition of COP portfolios. We introduced new metrics and studied how CSP portfolio solvers might be adapted to deal with COPs. In particular, we addressed the problem of boosting optimization by exploiting the sequential cooperation of different COP solvers;

- we finally reaped the benefits of our work for developing `sunny-cp`: a portfolio solver aimed at solving a generic CP problem, regardless of whether it is a CSP or a COP. To the best of our knowledge, `sunny-cp` is currently the only sequential portfolio solver able to solve generic CP problems.

Some of the contributions of this thesis have already been published. Most of the work in Chapter 4 has been published in [7, 8, 10]. A journal version of [7] is currently under revision. Chapter 5 mostly report the papers published in [9, 12]. A journal version of [9] is currently under revision. Chapter 6 presents an extended version of [11]. Papers [7, 8, 10, 9, 11] are joint works with Dr. Jacopo Mauro and Prof. Maurizio Gabbrielli, in which I contributed as primary author. Paper [12] was co-written with Prof. Peter J. Stuckey.

Chapter 2

Constraint Programming

"Constraint Programming represents one of the closest approaches Computer Science has yet made to the Holy Grail of programming: the user states the problem, the computer solves it."

Eugene C. Freuder.

Constraint programming (CP) is a declarative paradigm wherein relations between different entities are stated in form of constraints that must be satisfied. The goal of CP is to model and solve *Constraint Satisfaction Problems* (CSPs) as well as *Constraint Optimization Problems* (COPs) [159]. A CSP consists of three key components:

- a finite set of *variables*, i.e., entities that can take a value in a certain range;
- a finite set of *domains*, i.e., the possible values that each variable can take;
- a finite set of *constraints*, i.e., the allowed assignments to the variables.

Solving a CSP means finding a solution, i.e., a proper assignment of domain values to the variables that satisfies all the constraints of the problem. COPs can be instead regarded as generalized CSPs where we are interested in finding a solution that minimizes (or maximizes) a given *objective function*. The resolution of these problems is performed by software agents called *constraint solvers*. Henceforth, if not further specified, with the term "*CP problem*" we will refer to either a CSP or

a COP. Analogously, the term "*CP solver*" will refer to a constraint solver able to solve both CSPs and COPs.[1]

CP appeared in the 1960s in systems such as Sketchpad [70], and core ideas such as arc and path consistency techniques were proposed and developed in the 1970s. The real landmark of CP was in the 1980s with the coming of *Constraint Logic Programming* (CLP), a form of CP which embeds constraints into logic programs [20]. The first implementations of CLP were Prolog III, CLP(R), and CHIP. Another paradigm for solving hard combinatorial problems that has its foundations in logic programming is the *Answer-Set Programming* (ASP) [128]. Apart from logic programming, constraints can be mixed with other paradigms such as functional and imperative. Usually, they are embedded within a specific programming language or provided via separate software libraries.

Constraint Programming combines and exploits a number of different fields including for example Artificial Intelligence, Programming Languages, Combinatorial Algorithms, Computational Logic, Discrete Mathematics, Neural Networks, Operations Research, and Symbolic Computation. The applications of CP cover different areas: scheduling and planning are probably the most prominent ones, but constraints may also be considered for problems of configuration, networking, data mining, bioinformatics, linguistics, and so on. This chapter provides an overview of some of the main concepts and techniques used in the CP field.

2.1 Constraint Satisfaction Problems

In this section, after giving the basic notions of CSP and some examples, we provide an overview of the most common domain and the main solving techniques.

[1] Note that the borderline between CSP and COP solvers is fuzzy. Indeed, since a COP is a generalization of a CSP, a solver able to solve COPs can also solve CSPs. However, a CSP solver can solve a COP by enumerating and ranking every solution it finds. We however maintain this categorization since the different nature and solving techniques between CSPs and COPs.

Definition 2.1 (CSP) *A **Constraint Satisfaction Problem** (CSP) is a triple* $\mathcal{P} := (\mathcal{X}, \mathcal{D}, \mathcal{C})$ *where:*

- $\mathcal{X} := \{x_1, x_2, \ldots, x_n\}$ *is a finite set of **variables**;*
- $\mathcal{D} := D_1 \times D_2 \times \cdots \times D_n$ *is a n-tuple of **domains** such that* D_i *is the domain of variable* x_i *for each* $i = 1, 2, \ldots, n$*;*
- $\mathcal{C} := \{C_1, C_2, \ldots, C_m\}$ *is a finite set of **constraints** with arity* $0 \leq k \leq n$ *defined on subsets of* $\mathcal{X}$*. More formally, if* $C \in \mathcal{C}$ *is defined on a subset of variables* $\{x_{i_1}, x_{i_2}, \ldots, x_{i_k}\} \subseteq \mathcal{X}$ *then* $C \subseteq D_{i_1} \times D_{i_2} \times \cdots \times D_{i_k}$*.*

Definition 2.2 (Satisfiability) *Let* $\mathbf{d} := (d_1, d_2, \ldots, d_n) \in \mathcal{D}$ *be a n-tuple of domain values. It is said that* $\mathbf{d}$ satisfies *a constraint* $C \in \mathcal{C}$ *defined on variables* $\{x_{i_1}, x_{i_2}, \ldots, x_{i_k}\}$ *if and only if* $(d_{i_1}, d_{i_2}, \ldots, d_{i_k}) \in C$*. A constraint* C *is* satisfiable *if and only if there exists at least a* $\mathbf{d} \in \mathcal{D}$ *that satisfies* C *(otherwise* C *is said* unsatisfiable*). A tuple* $\mathbf{d} \in \mathcal{D}$ *is a **solution** of a CSP if and only if* $\mathbf{d}$ *satisfies every* $C \in \mathcal{C}$*. If a CSP has at least a solution is said **satisfiable** (otherwise, is said **unsatisfiable**).*

Given a CSP $\mathcal{P} := (\mathcal{X}, \mathcal{D}, \mathcal{C})$ the goal is normally to find a solution of $\mathcal{P}$, that is, a n-tuple $(d_1, d_2, \ldots, d_n) \in \mathcal{D}$ of domain values that satisfies every $c \in \mathcal{C}$.[2] To better clarify the above definitions, consider the following examples.

Example 2.1 (Send More Money) *The problem "*Send More Money*" is a classical crypto-arithmetic game published in the July 1924 issue of Strand Magazine by Henry Dudeney. The objective is to unequivocally associate to each letter* $l \in \{S, E, N, D, M, O, R, Y\}$ *a digit* $d_l \in \{0, 1, \ldots, 9\}$ *so that the equation* $SEND + MORE = MONEY$ *is met. This problem can be mapped to a CSP* $\mathcal{P} := (\mathcal{X}, \mathcal{D}, \mathcal{C})$ *where:*

- $\mathcal{X} := \{x_1, x_2, \ldots, x_8\}$*, where the variable* x_i *corresponds to the i-th element of the vector* $\mathbf{x} = \langle S, E, N, D, M, O, R, Y \rangle$*;*

[2] Sometimes the goal might also be to find all the solutions of the problem.

- $\mathcal{D} := D_1 \times D_2 \times \cdots \times D_8 = \{0, 1, \ldots, 9\} \times \{0, 1, \ldots, 9\} \times \cdots \times \{0, 1, \ldots, 9\}$;
- $\mathcal{C} := \{C_1, C_2, C_3\}$, *where:*
  - $C_1 := \{(d_1, d_2, \ldots, d_8) \in \mathcal{D} \mid d_1 > 0 \wedge d_5 > 0\}$,
    i.e., the most significant digits S and M must not be zero;
  - $C_2 := \{(d_1, d_2, \ldots, d_8) \in \mathcal{D} \mid d_i \neq d_j \text{ for } 1 \leq i < j \leq n\}$,
    i.e., all the variables in $\mathcal{X}$ must be distinct;
  - $C_3 := \{(d_1, d_2, \ldots, d_8) \in \mathcal{D} \mid 1000d_1 + 100d_2 + 10d_3 + d_4 + 1000d_5 + 100d_6 + 10d_7 + d_2 = 10000d_5 + 1000d_6 + 100d_3 + 10d_2 + d_8\}$,
    i.e., the equation $SEND + MORE = MONEY$ must be satisfied.

This problem is satisfiable: its (unique) solution is $\mathbf{d} = (S, E, N, D, M, O, R, Y) = (9, 5, 6, 7, 1, 0, 8, 2)$.

Example 2.2 (n-Queens) *The n-Queens problem consists in positioning $n > 3$ queens on a chessboard $n \times n$ in such a way that none of them can attack each other. This problem can be modelled by a CSP $\mathcal{P} := (\mathcal{X}, \mathcal{D}, \mathcal{C})$ with:*

- $\mathcal{X} := \{x_1, x_2, \ldots, x_n\}$, *where $x_i = j$ if and only if the i-th queen is placed on the j-th column of the chessboard. Note that since a variable can assume only one value, this also implies that it is not possible to place two queens on the same row;*
- $\mathcal{D} := D_1 \times D_2 \times \cdots \times D_n = \{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\} \times \cdots \times \{1, 2, \ldots, n\}$;
- $\mathcal{C} := \bigcup_{1 \leq i,j \leq n} \{C'_{ij}, C''_{ij}\}$ *where:*
  - $C'_{ij} := \{(d_1, d_2, \ldots, d_n) \in \mathcal{D} \mid d_i \neq d_j\}$, *i.e, no queens on the same column;*
  - $C''_{ij} := \{(d_1, d_2, \ldots, d_n) \in \mathcal{D} \mid |d_i - d_j| \neq i - j\}$ *i.e, no queens on the same diagonal.*

For example, if $n = 4$ there are two solutions: $\mathbf{d_1} = (2, 4, 1, 3)$ and $\mathbf{d_2} = (3, 1, 4, 3)$. However, since they differ only by a rotation on the chessboard, these solutions can actually be seen as a single one. Indeed, in the n-Queens problem there is plenty of *symmetries*, i.e., solutions that are identical up to symmetry operations (rotations and reflections). Symmetries occur naturally in many problems, and it is very important to deal with them to avoid wasting time to visit symmetric solutions, as well as parts of the search tree which are symmetric to already visited parts. Two common types of symmetry are variable symmetries (which act just on variables), and value symmetries (which act just on values) [43]. One simple but highly effective mechanism to deal with symmetry is to add constraints which eliminate symmetric solutions [47]. An alternative way is modify the search procedure to avoid visiting symmetric states [59, 76, 158, 63].

2.1.1 CSP on Finite Domains

Constraints can be of different types according to the domain of the variables involved. For example, if p, q, r are Boolean variables we could express constraints of the form $r \equiv p \vee q$ or $p \wedge q \rightarrow r$. If n is an integer variable and X, Y are set variables, we could consider a constraint of the form $|X \cap Y| \leq n$ which constrains X and Y to have at most n common elements. Although in theory there is no limitation on the domains of the variables (e.g., they could be discrete, continuous, or symbolic) the most successful CP applications are based on *Finite Domains* (FD), i.e., the domain of the variables has finite cardinality. This section provides an overview of the most widely used FD in Constraint Programming.

2.1.1.1 Boolean Variables

In CSPs with only Boolean variables, each variable can be set to either *true* or *false*. The most common class of Boolean CSPs is represented by *(Boolean, Propositional) Satisfiability problems*, better known as SAT problems [87]. In these problems the goal is to determine if there exists an interpretation (i.e., an assignment of Boolean

values to the variables) that satisfies a given Boolean formula.

Historically, the SAT problem was the prototype and one of the simplest NP-complete problems [45]. It attracted a lot of attention throughout the years, and nowadays is probably the area of greatest influence in the context of satisfiability problems. SAT solving is commonly used in many real life problems such as automated reasoning, computer-aided design and manufacturing, machine vision, database, robotics, integrated circuit design, computer architecture design, and computer network design. Its widespread use has fostered the dissemination of a large number of different solvers [81], which success is probably due to the combination of different techniques like nogoods, backjumping, and variable ordering heuristics [142]. In particular, as we will see below, SAT solving is the CSP branch in which *portfolio approaches* have been grown more. Starting from 2002, international SAT solver competitions [111] take place in order to evaluate the performance of different solvers on extensive benchmarks of real case, randomly generated and hand-crafted instances defined in the *Dimacs* standard format [112]. It is worth noting that, although technically SAT problems are simplified CSPs, in the literature there is a clear distinction between them and the "generic" CSPs (which typically have integer domains). This is because of the different nature between SAT solving and other approaches like FD solving or Linear Programming. Henceforth, if not better specified, with the term CSP we will refer only to generic CSPs, excluding from such categorization the SAT problems.

There is plenty of SAT variants and related problems like 2-SAT, 3-SAT, HORN-SAT, XOR-SAT. In particular, the *Maximum Satisfiability* problem (MAX-SAT) [21] is the problem of finding an interpretation that satisfies the maximum number of clauses. A well-known extension of SAT is *Satisfiability Modulo Theories* (SMT) [19] that enriches SAT formulas with linear constraints, arrays, all-different constraints, uninterpreted functions, and so on. These extensions typically remain NP-complete, but efficient solvers are developing to handle such types of constraints. The SAT problem becomes harder if we allow both existential $\exists$ and universal $\forall$ quantification: in this case, it is called *Quantified Boolean Formula* problem (QBF) [35]. In

particular, the presence of the $\forall$ quantifier in addition to $\exists$ makes the QBF problem PSPACE-complete.

2.1.1.2 Integer Variables

For most of the problems is more natural to model a CSP with integer variables belonging to a finite range (e.g., $[-1..5] = \{-1, 0, \ldots, 5\}$) or to a disjoint union of finite ranges (e.g., $[2..8] \cup [10..20]$).[3] Usually, a CSP over integers supports some standard basic constraints (like $=, \neq, <, \leq, >, \geq, +, -, \cdot, /, \ldots$) together with *global constraints*. A global constraint [159] can be seen as a constraint over an arbitrary sequence of variables. The most common global constraints usually come with specific algorithms (the so-called *propagators*) that may help to solve a problem more efficiently than decomposing the global constraint into basic relations. This feature is perhaps one of the main strengths of FD solvers when solving huge combinatorial search problems. The canonical example of a global constraint is the *all-different* constraint. An all-different constraint over a set of variables states that the variables must be pairwise different. This constraint is widely used in practice and because of its importance is offered as a built-in constraint in the large majority of commercial and research-based constraint programming systems. In fact, although from a purely logic point of view the semantic of *all-different*$(x_1, x_2, \ldots, x_n)$ is the same of the logic conjunction $\bigwedge_{1 \leq i,j \leq n} x_i \neq x_j$, the use of a specific propagator can make the resolution dramatically more efficient. Other examples of widely applicable global constraints are the *global cardinality constraint* (gcc) [156] or the *cumulative* constraint [5]. An exhaustive list of global constraints can be retrieved at [24].

Differently from SAT field, in CSP context there are fewer and less stable solvers (e.g., see the number of entries in SAT solver competitions [25] w.r.t. CP solver competitions [110, 139]). It should be noted that SAT problems are much easier to encode: a CSP may contain more complex constraints (e.g., global constraints

[3]Note that in the literature the term "FD solver" refers almost always to a solver operating on integer variables, possibly equipped with additional extensions (e.g., finite sets of integers). Moreover, sometimes the terms "FD solver" and "CP solver" are used as synonyms.

like *bin-packing* or *regular*). Moreover, at the moment the CP community has not yet agreed on a standard language to encode CSP instances. The XML-based language XCSP [161] was used to encode the input problems of the *International CSP Solver Competition* (ICSC) [109]. XCSP is still used but, since the ICSC ended in 2009, its spread over recent years has been restricted. At present, the *de-facto* standard for encoding (not only) CSP problems is *MiniZinc* [146]. MiniZinc is currently the most used, supported, and general language to specify CP problems. It supports also optimization problems and is the source format used in the *MiniZinc Challenge* (MZC) [176], which is nowadays the only international competition for evaluating the performances of CP solvers. MiniZinc is a medium-level language that reduces the overall complexity of the higher-level language *Zinc* [135] by removing user-defined types, various coercions, and user-defined functions. Unlike XCSP, MiniZinc provides the separation between model and data, is not restricted to integers, it allows if-then-else constructs, arrays, loops, varied global constraints, and more recently it added new features like user-defined functions and option types. *FlatZinc* [22] is a low-level language which is mostly a subset of MiniZinc. It is designed for translating a general model into a specific one that has the form required by a particular solver. Indeed, starting from a solver-independent MiniZinc model every solver can produce its own FlatZinc model by using solver-specific redefinitions. Apart from MiniZinc, other solver-independent modelling languages are ESRA [62], Essence [68], and OPL [180].

2.1.1.3 Other Variables

In the last years, researchers have given special attention to set variables and constraints. Many complex relations can be expressed with set constraints such as set inclusion, union, intersection, disjointness, cardinality. An example of a logic-based language for set constraints is CLP($\mathcal{SET}$) [55]. It provides very flexible and general forms of sets, but its effectiveness is hindered by its solving approach, which is strongly non-deterministic. More recently, other approaches were proposed to deal with finite sets of integers. Common ways to represent the domain of a set variable

are the *subset-bounds* representation [17] and the *lexicographic-bounds* representation [163]. A radically different approach based on the *Reduced Ordered Binary Decision Diagrams* (ROBDDs) is proposed in [95], an integration between BDD set solvers and SAT solvers is described in [96], and a framework for combining set variable representation is shown in [28]. Note that some FD solvers prefer to not use set constraints, since it is possible to get equivalent formulations by making use of only binary or integer variables.

With regard to the domain of rational and real numbers, some approaches have been proposed especially in the context of CLP. For example, CLP($\mathcal{Q}, \mathcal{R}$) system [99] was developed to deal with linear (dis)equations over rational and real numbers. A separate mention concerns instead the floating-point domain. Solving constraints over floating-point numbers is a critical issue in numerous applications, notably in program verification and testing. Albeit the floating-point numbers are a finite subset of the real numbers, classical CSP techniques are here ineffective since the huge size of the domains and the different properties of floating-point numbers w.r.t. real numbers. In [137] a solver based on a conservative filtering algorithm is proposed. In [32] the authors addressed the peculiarities of the symbolic execution of programs with floating-point numbers, while [23] proposes to use Mixed Integer Linear programming for boosting local consistency algorithms over floating-point numbers. The flexibility and generality of the CSP framework makes however possible its extension to non-standard domains. For example, CSPs have also been defined over richer data types like multi-sets (or bags) [117, 182], graphs [54], strings [80] and lattices [61].

2.1.2 CSP Solving

As mentioned earlier, solving a CSP means assigning to each variable a consistent value of its domain. Intuitively, a trivial technique could be the systematic exploration of the solutions space. This approach, also referred as *"Generate & Test"*, is based on a simple idea: a complete assignment of values to variables is iteratively generated until a (possible) solution is found. Obviously, despite this algorithm

guarantees the completeness of the search on finite domains, it becomes intractable very quickly when increasing the problem size. There are two main ways to improve the efficiency of this approach: using a "smart" assignments generator, so as to minimize the failures in the test phase; and merging the generator with the tester, which is the way of CSP solving. In the last case, the consistency is tested as soon as variables are instantiated by using a *backtracking* approach [136].

Backtracking is a general approach that tries to build a solution by incrementally exploring the branches of the search space in order to extend the current partial solution (initially empty). When an inconsistency is detected, the algorithm abandons that path and backtracks to a consistent node, thus eliminating a subspace from the Cartesian product of the domains. Despite this pruning allows to improve the Generate & Test approach, the running complexity of backtracking is still NP-hard for most non-trivial problems. The three major drawbacks of standard backtracking are: thrashing (i.e., repeated failures due to the same reason), redundant work (i.e., the conflicting values of variables are not remembered), and late detection (i.e., the conflict is not detected before it really occurs). We now present an overview of some of the major enhancements of the backtracking approach. A formal and detailed descriptions of the following methods is outside the scope of this thesis: for more details we refer the reader to the corresponding references.

2.1.2.1 Consistency techniques

A well-known approach for solving CSPs is based on *consistency* techniques. The basic consistency techniques are based on the so called constraint (hyper) graph —sometimes called constraints network— where nodes correspond to variables and (hyper) edges are labelled by constraints.

Several consistency notions are reported in the literature [125, 129]. Let us fix a problem $\mathcal{P} := (\mathcal{X}, \mathcal{D}, \mathcal{C})$. The simplest consistency technique is the *node-consistency* (NC), which requires that for every variable $x_i \in \mathcal{X}$, every unary constraint C on $\{x_i\}$, every $d \in D_i$, we have that $d \in C$. The most widely used consistency technique is called *arc-consistency* (AC). Formally, $\mathcal{P}$ is arc-consistent if for every

pair of variables x_i, x_j, every binary constraint C on $\{x_i, x_j\}$, every value $d \in D_i$, there exists a value $d' \in D_j$ such that $(d, d') \in C$. Even more invalid values can be removed by enforcing the *path-consistency* (PC). Path-consistency requires that for every triplet of variables x_i, x_j, x_k, every C on $\{x_i, x_j\}$, every C' on $\{x_i, x_k\}$, and every C'' on $\{x_j, x_k\}$, if $(d, d') \in C$ then it exists a $d'' \in D_k$ such that $(d, d'') \in C'$ and $(d', d'') \in C''$.

It can be shown that the above consistency techniques are covered by a general notion of (strong) *k-consistency* [66]. Indeed, NC is equivalent to strong 1-consistency, AC to strong 2-consistency, and PC to strong 3-consistency. Algorithms exist for making a constraint graph strongly k-consistent for $k > 2$, but in practice they are rarely used because of efficiency issues. Note that virtually all the consistency algorithms are not complete: in other terms, meeting a given notion of consistency does not imply that the CSP is really consistent. This is because achieving the completeness can be computationally very hard: from the practical perspective it is preferable to settle for relaxed forms of consistency that do not eliminate the need for search in general, but however allow to remove efficiently a significant amount of inconsistencies.

2.1.2.2 Propagation and Search

Even if both systematic search and consistency techniques might be used alone to completely solve CSPs, this is rarely done: a combination of both approaches is a more common way of solving. According to a given notion of local consistency, for each different kind of constraint suitable agents called *propagators* are responsible to remove the inconsistent values in order to meet such consistency notion. A local removal may trigger other propagators which in turn may remove values from other domains (e.g., consider the CSP in the Example 2.3).

Example 2.3 *Consider a CSP $\mathcal{P}$ with three variables x, y, z having domains $D_x := [0..1]$, $D_y := [-6..1]$, $D_z := \{1, 5\}$ and with two constraints $\{x < y, y \neq z\}$. $\mathcal{P}$ is node-consistent, but not arc-consistent. For example, if $x = 1$ then no value of D_y can satisfy the constraint $x < y$. In order to reach the AC, the propagator of $<$*

reasonably assigns the values 0 to x and 1 to y; this narrowing affects $y \neq z$, which becomes $1 \neq z$. Then, the propagator of $\neq$ removes 1 from D_z and consequently assign the value 5 to z.

The process of propagation is iterated until a *fix-point* is reached; this can happen when the domain of a variable becomes empty (i.e., the CSP is unsatisfiable) or no further values can be removed (i.e., the consistency notion is met). Unfortunately, since consistency techniques are usually incomplete, there is no guarantee that once a fix-point is reached the CSP is actually consistent. To overcome this limitation, it is necessary to perform a *search* in the solutions space for to verify if there exists a consistent assignment of values to variables. This process can be done either by considering all the variables of $\mathcal{X}$, or by performing and checking the assignments on a proper subset $\mathcal{L} \subset \mathcal{X}$ of so-called "labelled" variables. Different *search heuristics* can be adopted and combined to speed up the search (e.g., variable choice heuristics and value choice heuristics).

2.1.2.3 Nogood Learning and Lazy Clause Generation

In order to avoid some problems of backtracking, like thrashing and redundant work, *look-back* schemes like backjumping [71] or backchecking [94] were proposed. Typically look back schemes share the disadvantage of late detection of the conflict: they solve the inconsistency when it occurs but do not prevent the inconsistency to occur. Therefore, *look-ahead* schemes were proposed to prevent future conflicts. Forward checking is the easiest example of a look-ahead strategy.

A very effective technique for preventing conflicts and reducing the search space consists in learning conflicts during the search [51]. In the SAT context this is better known as *clause learning*. The development and the refinement of clause learning techniques has led to dramatic improvements for SAT solving [142]. The equivalent of clause learning in CSP field is called *nogood learning*. Nogoods are redundant constraints that in [51] are technically defined as variable assignments that are not contained in any admissible solution. They can be learned during search, stored, and used to prune further part of the search tree. In [115] the authors pointed out

that nogood learning in CSP has not been as successful as clause learning in SAT, and also proposed a generalization of standard nogoods.

There exists a lot of work proposing different techniques for encoding a CSP into a SAT problem [4, 104, 18, 178, 174]. A hybrid approach, called *Lazy Clause Generation*, is instead presented in [149]. Lazy clause generation combines the strengths of CP propagation and SAT solving. The key idea is to mimic the underlying rules of FD propagators by properly generating corresponding SAT clauses. The clause generation is "lazy" since it is not performed *a priori*, but it occurs during the search. This approach enables a strong nogood learning, able to detect and analyse the conflicts that occur during the search. Benefits of lazy clause generation on the RCPSP/max problem are shown in [168]. Moreover, the lazy clause generation solver *Chuffed* [78] has dominated the MiniZinc Challenges 2012–2014.

2.1.2.4 Local Search

The large size and the heterogeneous nature of real-world combinatorial problems make it sometimes impracticable the use of exact approaches. A possible workaround consists in using *Local Search* (LS) methods. LS methods are greedy approaches based on a simple and general idea: trying to improve a current "local" solution by moving from time to time toward a possibly better solution within a given neighbourhood. If there are not better solutions in the neighbourhood, it means that a local optimum was reached. To avoid getting stuck in a local optimum, several effective techniques can be applied.

In [65] different hybrid methods are reported for combining the efficiency of LS with the flexibility of CP paradigm. Some local search methods (e.g., [39, 49, 148, 152]) used CP as a way to efficiently explore large neighbourhoods with side constraints. Others, such as [40], used LS as a way to improve the exploration of the search tree.

In the particular context of the CSPs, a LS approach iteratively tries to improve an assignment of the variables until all the constraints are satisfied. The local search is therefore performed in the space $\mathcal{D}$ of the possible assignments, by means of a

proper evaluation function for measuring the quality of the assignments (e.g., in terms of the number of violated constraints).

Two main classes of local search algorithms exist. The first one is that of greedy or non-randomized algorithms. Well-known examples of greedy algorithms are the Hill Climbing [169] and the Tabu Search [79]. The main drawback of these algorithms concerns the possibility of getting stuck in a sub-optimal state. To overcome this problem, randomized LS algorithms has been devised. Examples of such random-walk algorithms are the the WalkSAT/GSAT [170] and the Simulated Annealing [181].

2.2 Constraint Optimization Problems

In many real-life applications we are not just interested in finding "a" solution but "the" optimal solution, or at least a good one. The quality of the solutions is usually measured by an application-dependent function called *objective function* which can represent a cost as well as a gain. The goal is no longer just finding a solution, but finding one that minimizes or maximizes the objective function. These kinds of problems are referred to as *Constraint Optimization Problems* (COPs)[4].

From now on, without loss of generality, we will always consider a COP as a minimization problem. Indeed, it is always possible to switch from a maximization problem to an equivalent minimization problem by simply negating the objective function. Formally, a COP can be defined as follows:

Definition 2.3 (COP) *A **Constraint Optimization Problem** (COP) is a quadruple $\mathcal{P} := (\mathcal{X}, \mathcal{D}, \mathcal{C}, f)$ where:*

- $\mathcal{P}' := (\mathcal{X}, \mathcal{D}, \mathcal{C})$ *is a CSP;*
- $f : \mathcal{D} \rightarrow \mathbb{R}$ *is the* objective function *of* $\mathcal{P}$.

[4] Sometimes a COP is also referred as a Combinatorial Optimization Problem [86, 167].

The goal is normally to find a solution of $\mathcal{P}'$ that minimizes f. Clearly, a COP is more general than a CSP (that can be instead regarded as a particular COP in which f is constant over $\mathcal{D}$). For instance, a solution $\mathbf{d}$ found by a COP solver can be *sub-optimal* (i.e., there exists at least a better solution $\mathbf{d}' < \mathbf{d}$). Moreover, a COP solver may find an optimal solution $\mathbf{d}^*$ without being able to prove its optimality.[5]

Well-known examples of COPs are for instance the Cutting-stock problem [90] (essentially reducible to the Bin-packing and Knapsack problems), the Vehicle Routing Problem (VRP) (introduced in [48] as a generalization of the Travelling Salesman Problem (TSP) [64]), and the Resource-Constrained Project Scheduling Problem (RCPSP) [34].

A widely used algorithm for solving COPs is called *Branch and Bound* (BB). This method was first proposed in [126] for discrete programming, and has become the most commonly used tool for solving combinatorial optimization problems. A BB procedure consists essentially in two steps. The *branching* step recursively splits the original problem into sub-problems or, in other terms, splits the search tree into sub-trees. The *bounding* step instead estimates the lower and upper bounds of the objective function f over each sub-problem. The key idea of BB algorithm is: if the lower bound for a sub-problem P_1 is greater than the upper bound for another sub-problem P_2, then P_1 can be safely discarded from the search (*pruning*).

In the rest of the section we focus in particular on two aspects of constrained optimization: the so-called Soft Constraints, and the Operations Research techniques for dealing with constrained optimization.

2.2.1 Soft Constraints

When a large set of constraints needs to be solved, it is not unlikely that there is no way to satisfy them all: the problem is said to be *over-constrained*. Moreover, it

[5]Normally, for CP solvers the codomain of f is a finite subset of $\mathbb{N}$. However, in case of non-finite domains, a COP solver should also be able to prove the *unboundedness* of a problem. For example, let us consider a simple COP defined by $\mathcal{P} := (\{x\}, \mathbb{Z}, \emptyset, f(x) := -x)$. In this case $\mathcal{P}$ is unbounded, since $f(x)$ is unbounded in $\mathbb{Z}$.

could be that several solutions are equally optimal. This can be caused by an inappropriate modelling: constraints are used to formalize desired properties rather than preferences (i.e., conditions whose violation should be avoided as far as possible). *Soft constraints* provide a way to model the preferences. In this section, inspired by [159], we provide an overview of the most important classes and techniques of soft constraints.

2.2.1.1 Fuzzy Constraints and Weighted CSPs

There are many classes of soft constraints. The *Fuzzy Constraints* approach is based on the Fuzzy Set Theory [57, 56]. Fuzzy constraints map the preferences in a range between 0 (total rejection) and 1 (complete acceptance). The preference of a solution is computed by taking the minimal preference over the constraints. This may seem counter-intuitive in some scenarios, but it is instead more natural in others. For example, in critical settings like medical applications we would like to be as cautious as possible. Probabilistic constraints [165] and fuzzy lexicographic constraints [60] are variants of classical fuzzy constraints.

In other contexts we are more interested in the damages we get by not satisfying a constraint, rather than in the advantages we obtain when we satisfy it. In *Weighted Constraint Satisfaction Problems* (WCSPs) each constraint is provided with a weight representing the penalty to be paid when such constraint is violated. An optimal solution is therefore a solution that minimizes the sum of the weights of the violated constraints. If all the weights are set to 1, we get the MAX-CSP problem [67]: similarly to the MAX-SAT problem, here the goal is maximizing the number of satisfied constraints. Weighted constraints are among the most expressive soft constraint frameworks, since fuzzy constraint problems can be efficiently reduced to weighted constraint problems [166].

2.2.1.2 Formalism and Inference

The literature contains at least two general formalisms to model soft constraints: *semiring-based* constraints [29, 30] and *valued constraints* [166]. Semiring-based

constraints rely on a simple algebraic structure which is very similar to a semiring. Valued constraints depend instead on a positive totally ordered commutative monoid and use a different syntax w.r.t. semiring-based constraints. However, if we assume preferences to be totally ordered [31], they have the same expressive power. Soft constraint problems are as expressive, and as difficult to solve, as constraint optimization problems. Indeed, given any soft constraint problem we can always build a COP with the same solution ordering, and vice versa.

Inference in soft constraints reflects the same basic notions of classical constraints. *Bucket elimination* (BE) [52, 53] is a complete inference algorithm which is able to compute all the optimal solutions of a soft constraint problem. The high memory cost is the main drawback of using BE in practice. Because complete inference can be extremely time and space intensive, it is often more interesting to use simpler but more efficient techniques of soft constraint propagation.

2.2.2 Optimization and Operations Research

In a nutshell, *Operations Research* (OR, also called Operational Research) is the discipline of applying advanced analytical methods to help make better decisions. Operations Research originated in the efforts of military planners during World War II, and subsequently largely adopted for civilian purposes in a huge variety of fields including business, finance, logistics, and government. OR encompasses a wide range of problem-solving techniques and methods applied in the pursuit of improved decision-making and efficiency, such as simulation, mathematical optimization, queueing theory and other stochastic-process models, Markov decision processes, econometric methods, data envelopment analysis, neural networks, expert systems, decision analysis, and the analytic hierarchy process.[6] In particular, COPs are well studied and used in practice in areas such as services, logistics, transports, economics, and in many other industrial applications. Operations research has proved to be useful for modelling problems of planning, routing, scheduling,

[6]From `http://en.wikipedia.org/wiki/Operations_research`.

assignment, and design. In this section we provide an overview of the classical OR optimization methods and a comparison between CP and OR techniques.

2.2.2.1 Linear Programming

Linear programming (LP) is a general OR optimization method in which both the constraints and optimization function are linear. The canonical form of a LP problem is:

$$\begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & Ax \leq b, \quad x \geq 0 \end{array}$$

where $x \in \mathbb{R}^n$ is the vector of the variables to be assigned, $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$ are vectors of known coefficients (c^{T} is the transpose of c) while $A \in \mathbb{R}^{m \times n}$ is the matrix of the constraints coefficients. The inequalities $Ax \leq b$ are constraints that specify a convex polyhedron (the feasible region) over which the objective function $f(x) = c^T x$ has to be maximized.

Every LP problem (or linear program), referred to as a *primal* problem, can be converted into a corresponding *dual* problem, which provides an upper bound to the optimal value of the primal problem [33]. Note that the dual of a dual linear program is the original primal linear program. Given the above definition of primal problem, the corresponding dual is:

$$\begin{array}{ll} \text{minimize} & b^T y \\ \text{subject to} & A^T y \geq c, \quad y \geq 0 \end{array}$$

The theory of the duality shows some interesting properties (e.g., the duality theorems) and it is also exploited by the *simplex algorithm* [144]. This method, devised by George Dantzig in 1947, makes use of the concept of simplex (i.e., a polytope of $n+1$ vertices in n dimensions) for solving LP programs. Other effective techniques for solving LP problems are instead based on *interior point* methods [153].

The general LP framework can be specialized according to the variables domain. For instance, if all of the variables are required to be integers it is called an *Integer*

Linear Programming (ILP) problem. In contrast to LP, which can be solved efficiently in the worst case, ILP problems are in many practical situations (those with bounded variables) NP-hard. A special case of ILP where variables are constrained to be either 0 or 1 is the *Binary Integer Programming* (BIP, also called 0-1 integer programming). Despite the binary domain of the variables, this problem is also classified as NP-hard. If only some of the variables are required to be integers, then the problem is called a *Mixed Integer Programming* (MIP) problem. These are generally NP-hard because they are even more general than ILP programs. However, despite the NP-hardness, some important subclasses of ILP and MIP problems are efficiently solvable. In addition to BB, other advanced algorithms for solving LP problems include for instance the cutting-plane method and the column generation.

2.2.2.2 Nonlinear Programming

In contrast to LP, in *Nonlinear Programming* (NLP) problems some of the constraints or the objective function are nonlinear. Formally, an NLP program has the following form:

$$\begin{array}{rll} \text{minimize} & f(x) & \\ \text{subject to} & g_i(x) = 0 & \text{for } i = 1, 2, \ldots, m \\ \text{and} & h_j(x) \geq 0 & \text{for } j = m+1, m+2, \ldots, n \end{array}$$

where $n \geq 0$ is the total number of constraints of the problem, $0 \leq m \leq n$ is the number of equalities and at least one function in $\{f, g_1, g_2, \ldots, g_m, h_{m+1}, h_{m+2}, \ldots, h_n\}$ is nonlinear.

A well-known subclass of NLP problems is constituted by the *Quadratic Programming* (QP) problems. A QP problem is the problem of optimizing a quadratic function subject to linear constraints. Formally, it can be formulated as:

$$\begin{array}{rl} \text{minimize} & \frac{1}{2}x^T Q x + c^T x \\ \text{subject to} & Ax \leq b \\ \text{and} & Ex = d \end{array}$$

where $x \in \mathbb{R}^n$ represents the vector of variables; $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, and $d \in \mathbb{R}^p$ are vectors of known coefficients while $A \in \mathbb{R}^{m \times n}$, $E \in \mathbb{R}^{p \times n}$ and $Q \in \mathbb{R}^{n \times n}$ are matrices of known coefficients (in particular, Q is symmetric).

A related programming problem, called *Quadratically Constrained Quadratic Programming* (QCQP), can be posed by adding quadratic constraints on the variables. For general QP problems a variety of methods are commonly used, including interior point, active set [143], augmented Lagrangian [44], conjugate gradient, gradient projection, extensions of the simplex algorithm [143]. QP is particularly simple when only equality constraints appear. If Q is a positive definite matrix, the ellipsoid method solves the problem in polynomial time [123].

Operating Research vs. Constraint Programming

Roughly speaking, Operating Research and Constraint Programming can be regarded as different approaches for solving hard combinatorial problems. Both of these techniques has strengths as well as weaknesses, for which reason it is not possible to determine which is the best technique to be adopted in general. As often happens in Computer Science, some algorithms work very well on a certain class of problems, but are ineffective for others. In these cases, often the best solution is to use a hybrid approach able to merge the strengths of the different algorithms. CP and OR have indeed complementary strengths: on the one hand CP provides an easy way to deal with inference methods, logic processing, high-level problem modelling and local consistency; on the other, OR works well with relaxation methods, duality theory, atomistic problem modelling, and global consistency. Consequently, in order to achieve better performances and solve large combinatorial problems, it has become natural try to integrate these two approaches and the links between the two communities have grown stronger in recent years [138].

The general advantages of CP consist in being better at sequencing and scheduling, in the more natural modelling, in the use of global constraints, and in a natural way to locally control the constraints. In contrast, CP paradigm is usually weaker when treating discrete and continuous variables as well as over-constrained and op-

timization problems. Moreover, this field is younger and less explored if compared to OR. Some experimental results show that a hybrid methodology allows to significantly outperform both CP and OR for various classes of problems (e.g., planning and scheduling-based problems) [100].

The emerging research field of the integration between OR techniques and CP is promising and stimulating. Some of the main challenges concern the interaction between the user and the solving process, the resolution of partially unknown or ill-defined problems, the processing of large scale over-constrained problems, and the improvement of the CP solving process, both in the constraints propagation and in the solution search [138].

Chapter 3

Portfolios of Constraint Solvers

"Multae Manus Onus Levant"[1]

It is well recognized in the field of Artificial Intelligence, but we could say in Computer Science in general, that different algorithms have different performance when solving different problems (even belonging to the same class). As pointed out also by the "No Free Lunch" theorems [184, 185] it is evident that a single algorithm can not be a *panacea* for all possible problems. Given a problem x and a collection of different algorithms $A_1, A_2, \ldots, A_m$, the *Algorithm Selection* (AS) problem basically consists in selecting which algorithm A_i performs better on x.

The AS problem was introduced by John R. Rice in 1976 [157]. An overall diagram of the (refined) model he proposed is depicted in Figure 3.1. Given an input problem x, a proper vector $f(x)$ of real valued *features* is first extracted from x. Features are essentially instance-specific attributes that characterize a given problem. Dealing with features is crucial in AS: the idea is selecting, via a proper selection mapping S, the best algorithm $A = S(f(x))$ for problem x on the basis of the feature vector $f(x)$. The notion of "best algorithm" is not self-contained but defined according to suitable metrics for measuring the algorithm performance. Formally, the performance of algorithm A on x is mapped by a performance function P to a measure space $p = P(A, x) \in \mathcal{R}^n$. The performance of algorithm A on x is then a measure $\|p\| \in \mathcal{R}$ obtained from $P(A, x)$ to be maximised or minimised.

[1] "Many hands lighten the load".

It is clear that this model has several degrees of freedom. For instance, how to properly define a problem space $\mathcal{P}$? What are the best features for $f(x)$? How to pick an algorithm $A \in \mathcal{A}$? Which metric is a reasonable to measure the performance of A on x?

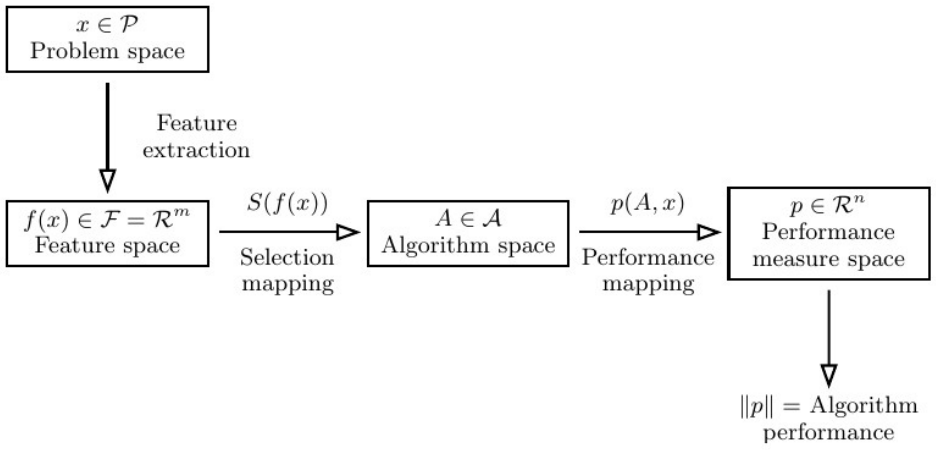


Figure 3.1: Refined model for the Algorithm Selection Problem (taken from [118]).

In this thesis we focus on what we could define as a particular case of AS: the *Algorithm Portfolio* [84] approach applied to CP solving. The boundary between Algorithm Selection and Algorithm Portfolios is fuzzy and these two related problems could be considered as synonyms. According to [118] definition, Algorithm Portfolios can be seen as particular instances of the more general AS framework in which the algorithm selection is performed case-by-case instead that in advance and a *portfolio* of algorithms corresponds to the algorithm space $\mathcal{A}$. In particular, within the CP context the algorithm space consists of a portfolio $\{s_1, s_2, \ldots, s_m\}$ of different CP solvers. We can thus define a *portfolio solver* as a particular constraint solver that exploits the synergy between the constituent solvers of its portfolio. When a new unseen problem p comes, the portfolio solver tries to predict which is (or which are) the best constituent solver(s) $s_1, s_2, \ldots, s_k$ $(k \leq m)$ for solving p and then runs such solver(s) on p.

The solver selection process is clearly a fundamental part for the success of a portfolio approach and is usually performed by exploiting *Machine Learning* (ML)

techniques. ML is a broad field that uses concepts from computer science, mathematics, statistics, information theory, complexity theory, biology and cognitive science in order to "construct computer programs that automatically improve with experience"[140]. In particular, *classification* is a well-known ML problem that, given a finite number of classes (or categories), consists in identifying to which class belongs each new observation. A classifier is essentially a function mapping a new instance —characterized by discrete or continuous features— to one class [140]. In supervised learning a classifier is defined on the basis of a training set of instances whose class is already known, trying to exploit such a knowledge to properly classify each new unseen instance.

The performance measure space for a (portfolio) solver depends instead on which kind of problems are considered. If the problem space consists of only CSPs, the evaluation is straightforward: the outcome of a CSP solver can be either "solved" (i.e., a solution is found or the unsatisfiability is proven) or "not solved" (i.e., the solver gives no answer). Things become trickier when COPs are considered, since a solver can provide sub-optimal solutions or even give the optimal one without proving its optimality. For a detailed discussion regarding different evaluation metrics for CP solvers we refer the reader to Chapter 4 for CSPs and Chapter 5 for COPs. In this chapter we want instead to introduce the main components that generally characterize a CP portfolio solver, namely: the dataset of CP instances used to make (and test) predictions, the constituent solvers of the portfolio, the features used to characterize each CP problem, and the techniques used to properly select and execute the constituent solvers.

3.1 Dataset

With the term *dataset* we will refer to what in the Rice's model is generically referred as the problem space $\mathcal{P}$. A dataset is a data sample that should be as exhaustive as possible, covering a significant number of problems belonging to different classes. Gathering an adequate dataset is fundamental to build and evaluate a prediction

model:[2] if the sample is not representative, it is hard to draw meaningful assessments. These difficulties had already been identified in [157]: many problem spaces are not well known and often a sample of problems is drawn from them to evaluate empirically the performance of the given set of algorithms.

Most portfolio approaches employ ML to learn and test a prediction model. In particular, the dataset often consists in the disjoint union of two problem spaces: a training set and a test set. A *training set* is a set of already known problems used to build the prediction model. The basic idea is to run each constituent solver on every problem of the training set, so as to learn the information relevant to the prediction. This process is also called training phase. A *test set* instead refers to a set of new, unseen problems used to evaluate the effectiveness of the formerly trained portfolio solver.

A well-known method for training and test a prediction model is the *k-fold cross validation* [15]. In a nutshell, this method consists in partitioning the dataset $\mathbb{D}$ in k disjoint folds $\mathbb{D}_1, \mathbb{D}_2 \ldots, \mathbb{D}_k$ such that $\mathbb{D} = \bigcup_{i=1,2,\ldots,k} \mathbb{D}_i$. In turn, for $i = 1, 2, \ldots, k$, one fold $\mathbb{D}_i$ is used as test set while the union $\bigcup_{j \neq i} \mathbb{D}_j$ of the remaining folds is used as training set. In order to avoid *overfitting* problems (arising from prediction models that adapt too well on the training data, rather than learning and exploiting a generalized pattern) it is possible to randomly repeat the folds generation more than once, then considering the average results over the repetitions. Another technique that can be effective to improve the performances accuracy consists instead in filtering the dataset according to certain heuristics [102].

All the dataset problems should be encoded in the same solver-independent language for being later processed by the constituent solvers. Unfortunately, as already mentioned in 2.1.1.2, CP community has not yet agreed on a standard language to express problem instances. If, on the one hand, the variety of languages allows for greater flexibility and freedom of modelling, on the other hand it also represents an obstacle for the definition and the standardisation of (portfolio) solvers. For

[2] With the term "prediction model" we refer to the set of data, knowledge, and algorithms required to predict and run the best solver(s) for solving a new CP problem.

instance, even if MiniZinc can be considered nowadays a *de-facto* standard, the biggest existing dataset of CSP instances we are aware is the one used in the ICSC 2008/09, which relies on XCSP language. To overcome this limitation we developed `xcsp2mzn`, a compiler from XCSP to MiniZinc language.

CSPLib [77] is a well-known library of CP problems that, however, are primarily described in the natural language.

3.2 Solvers

A portfolio solver is composed by a number of different constituent solvers. Clearly, each of them must support a common format in which problems are encoded. It is of course desirable to include in the portfolio state-of-the-art and bug-free solvers. Among the solvers that participated in the last MiniZinc Challenges, worth mentioning are Chuffed [78], OR-Tools [3], Opturion CPX [46], iZplus [2], Choco [1]. However, it is equally (if not more) important that the solvers are "complementary": they should be able to solve the greatest number of instances belonging to the most disparate classes of problems. In other terms, an important factor for the success of a portfolio is the *marginal contribution* [187] of the constituent solvers, where with marginal contribution of a solver S we mean the difference in performance between a portfolio solver including S and a portfolio solver excluding S.

An interesting aspect of portfolio solving concerns the size of a portfolio. As will be observed later, we experimentally verified that increasing the number of the constituent solvers does not necessarily imply an increase in performance of the portfolio. In fact, even when all the constituent solvers have a potentially positive marginal contribution, the solvers prediction could become inaccurate due to the presence of too many candidates solvers. This scalability issue has to be taken into account when designing a portfolio solver.

Note that a portfolio of solvers might be constituted by different parameter configurations of the same solver(s). This is particularly useful when dealing with highly parametrised solvers. The problem of automatic parameters tuning, also referred

as the *Algorithm Configuration* problem, has attracted some attention in recent years. This is because with increasing of the number of parameters it becomes very hard (even for experts) to manually tune the parameters configuration. Algorithm configurators are also used for building portfolios. For instance, Hydra [186] and ISAC [114] portfolio builders exploit automatic configurators like ParamILS [107], GGA [13] or SMAC [106]. Problems of automatic building and/or configuration of portfolios are however outside the scope of this thesis.

Although in this thesis we focus only on sequential portfolio solvers, we conclude this section by writing a few fords about parallel approaches on the same multicore machine (excluding massive approaches on cluster of machines). Having a finite portfolio, its parallelisation would seem a trivial issue: you only need to run in parallel all the solvers. Unfortunately, often the number of the constituent solvers exceeds the number of available cores. Furthermore, even assuming to have fewer solvers than cores, it is likely that —due to synchronization and memory consumption issues— running in parallel all the solvers on the same multicore machine is actually far from running the same solvers on different machines [162]. A naive multiprocessing approach could even result in wasting a huge amount of resources for little gains (e.g., see the examples of portfolios consisting of nearly 60 solvers in [147]). If we look at the results of the MiniZinc Challenge 2014 we notice that —despite the availability of eight logical cores— the performance of parallel solvers were not much better than those of sequential ones.

3.3 Features

Features are a key component in building an effective prediction model. They consist of collections of numeric attributes that characterize a given problem. In Rice's model (see Figure 3.1 on page 30) a feature vector $f(x) = \langle f_1, f_2, \ldots, f_m \rangle$ for each input problem x is extracted from a feature space $\mathcal{F} \subseteq \mathbb{R}^m$.

The choice of $f(x)$ is arbitrary and involves various distinctions. According to [118], a first discrimination can be made depending on the domain knowledge re-

quired to use the features. Indeed, some features may be so specific as to require a deep knowledge of the application domain in which they are embedded. Within the context of CP solving, some of the most common features —used for instance in [151, 154, 188]— include statistics on the variables, the domains, the (global) constraints, the objective function (for COPs). Note that all such features are called *static*, since they are purely syntactical and computed off-line by parsing the input problem. Conversely, *dynamic* features are computed on-line by retrieving information from short runs of a designated solver (e.g., the number of nodes explored or the number of propagations performed). The advantage of static features is that, unlike the dynamic ones, they are independent from the particular machine on which they are extracted. In contrast, dynamic features also allow to take into account some runtime information of the current problem. While some portfolios use an hybrid approach by combining both static and dynamic features (e.g., [151, 188]), other techniques rely on *on-line* features. Differently from dynamic features, on-line features are not extracted after a short run of a designed solver: they are computed directly during the search by monitoring the anytime performance of solvers [37, 38, 172].

An important step, which can dramatically improve the predictive accuracy, is the *feature selection*. Indeed, often not all the features are equally important: the presence of uninformative features might weaken the discriminating power of other more important features. For instance, features that are constants over the whole dataset do not provide any useful information. The approaches based on distance metrics like the *k-Nearest Neighbours* (k-NN) algorithm [6] typically scale the features values in a reduced range. Formally, if $f(x) = \langle f_1, f_2, \ldots, f_m \rangle$ is a feature vector, $[a, b]$ a range of values, and l_i, u_i are respectively the known lower and upper bound for feature f_i, then scaling a feature value f_i in a range $[a, b]$ means mapping the original value $f_i \in [l_i, u_i]$ to a scaled value $f'_i \in [a, b]$. In this thesis we consider only linear scaling such that $f'_i = a + (b - a)\dfrac{f_i - l_i}{u_i - l_i}$. The vector $f'(x) = \langle f'_1, f'_2, \ldots, f'_m \rangle$ is also called *normalized* feature vector.[3] Commonly choices

[3] This is an abuse of notation w.r.t. the standard definition of normalized vector, commonly

for $[a, b]$ range are $[0, 1]$ or $[-1, 1]$. There exists an extensive literature regarding the identification of the most significant features, see for example [127, 188, 154, 122, 119, 124]. It is noteworthy that feature selection is not a purely independent process, but is instead strictly related to the dataset and the solver selectors used. Although undoubtedly interesting, a depth analysis and evaluation of the features is not crucial part of this thesis, where we preferred to focus on other aspects.

Finally, note that the time needed to compute features must be taken into account, since it contributes to the total solving time of a given problem. Therefore, especially for runtime sensitive applications, all the features should be computed in a reasonable time. For example, retrieving features from the constraint graphs may be very time/space consuming.

3.4 Selectors

As pointed out in [118], there are many different ways to properly select one or more constituent solver(s). Note that, even if the prediction accuracy is of course welcome, in the CP context this requirement can be relaxed pragmatically. In a real setting selecting a solver which is not the best one for a given instance may be not so disadvantageous, provided that such solver is however able to solve that instance in a reasonable time. In other words, it might be sensible to prefer the maximization of the number of solved instances, eventually at the expense of the solving time minimization.

Among the different selection algorithms, a primary distinction can be done between the approaches that require to build an "explicit" prediction model and the so called *lazy* approaches [118]. Lazy approaches in fact do not learn an explicit model, but just use the set of training examples as a case base. For new problems, a set of the $k \geq 1$ closest problems in the training set is determined (via k-NN algorithm [6]) and decisions are made accordingly. However, note that also for lazy

defined as $\frac{f(x)}{\|f(x)\|}$ where $\|f(x)\|$ is the norm of $f(x)$.

approaches is necessary to keep track of the solvers execution on every instance of the training set. In order to reduce this effort, an interesting direction is shown in [173] where the prediction model is built by using only short runs of the constituent solvers on the training instances.

Some lazy approaches have been studied and evaluated, see for instance [154, 151, 147, 72, 164]. Avoiding (or at least lightening) the training phase can be advantageous in terms of simplicity and flexibility: it facilitates the treatment of new incoming problems, solvers and features. Among lazy approaches, it is worth mentioning the CSP portfolio solver CPHydra [151] which won the ICSC 2008. Another rather promising lazy approach is our SUNNY, that we discuss later on in Section 4.2.1.

However, empirical evidences prove that non-lazy techniques can be very effective. The main drawback of these approaches is that the training phase is often sophisticated, not flexible, and computationally expensive. For instance, a state-of-the-art SAT portfolio solver like SATzilla [190] relies on a weighted Random Forest algorithm while 3S [113], ISAC [114], and CHSC [131] during their off-line phase cluster the instances of the training set.

A further distinction between can be made between algorithms that select just one solver and those that schedule more solvers within a given time window. Even though at a first glance running in sequence more than one solver might seem not very efficient, yet this technique has practical advantages. One of the main empirical reasons behind this choice lies in the *heavy tailed* nature of solving. Indeed, many combinatorial problems are very easy for some solvers but, at the same time, almost impossible to solve for others. Moreover, in case a solver is not able to solve an instance quickly, it is not unlikely that it will take a huge amount of time to solve it. Therefore, selecting and scheduling a subset of the constituent solvers of the portfolio instead of trying to predict the "best" one for a given instance appears to be a good idea. This strategy ideally allows one to solve the same amount of problems, minimizing however the risk of choosing the wrong solver. In addition, executing more than one solver enables the communication of potentially relevant

information such as bounds, cuts or nogoods.

3.5 Related Work

We conclude this chapter by providing an overview on the related works. For comprehensive surveys about algorithm selection and runtime prediction, we refer the interested reader to [171, 108]. There are also several doctoral dissertations related to the AS problem, namely: [175, 105, 36, 69, 58, 120, 130].

Among the most promising sequential approaches in Chapter 4 we present and evaluate in detail CPHydra [151], 3S [113], and SATZilla [190] that were gold medallist respectively in the ICSC 2008, the SAT Competition 2011, and the SAT Challenge 2012. In the same chapter we present ISAC [114] and SUNNY (see in particular 4.2.1).

A new SAT portfolio was proposed in [131]. This approach improves 3S since it runs a static schedule for 10% of the available time and then executes for the remaining time a solver selected with a Cost-Sensitive Hierarchical Clustering (CSHC). This solver won two gold medals in the SAT competition 2013. The CSHC approach was not evaluated in this work since its code was not publicly available.

Apart from CPHydra and SUNNY, there are only few other approaches that can deal with CSPs. In [14] ML techniques are used to enhance the performances of a single CSP solver by dynamically adapting its search heuristics. These works list an extensive set of features to train and improve the heuristics model through Support Vector Machines. Proteus is a brand new CSP portfolio solver introduced in [104]. It does not purely rely on CSP solvers, but may decide to convert a CSP into a SAT problem by selecting an appropriate encoding and a corresponding SAT solver.

Recalling that in this work we focus only on sequential approaches, we would however mention some portfolio-based parallel SAT solvers like ManySAT [92], PeneLoPe [16] and ppfolio [160]. A parallel CP portfolio solver built on top of Numberjack platform [98] attended the MiniZinc Challenge 2013.

The ASP field presents a lot of similarities w.r.t. the state of research in the CSP field. Despite the development of portfolios is not as well studied as in SAT, there exist some portfolio approaches. Worth mentioning is claspfolio [101] that also won different tracks of the 2009 and 2011 ASP competitions.

With regard to COP portfolios, we can say that many of them are are mostly developed just for some specific COPs like Knapsack, Most Probable Explanation, Set Partitioning, and Travel Salesman Problem [108, 88, 179]. In [189, 114] automated algorithm configurators are used to boost the solving process. More details about COP portfolio solvers are provided in Chapter 5.

Finally, a number of tools are being developed in order to improve portfolio solvers usability: snappy [164] is a simple and training-less algorithm portfolio which relies on a k-NN prediction mechanism; LLAMA (Leveraging Learning to Automatically Manage Algorithm) [121] is instead a framework that facilitates the exploration of different portfolio techniques on any problem domain, by supporting the most common solver selectors and possibly combining them. To the best of our knowledge, as shown in Chapter 6, `sunny-cp` is currently the only sequential portfolio solver able to solve generic CP problems encoded in MiniZinc.

Chapter 4

Portfolios of CSP Solvers

"Satis Quod Sufficit"[1]

Within the SAT solving field, or at least in the context of SAT competitions, portfolio approaches appear to be quite widespread and effective. For instance, SATzilla [188, 190] has collected several gold medals in the SAT Competitions 2007, 2009 and the SAT Challenge 2012. 3S [113] won 7 medals in SAT Competition 2011, while CHSC [131] won the gold medal in the Open Track of SAT Competition 2013.

Unfortunately, the dissemination of portfolios within the CSP field is much more limited. There are several reasons for this gap between CSP and SAT. First of all, the CSP solving field is trickier: constraints can be arbitrarily complex (e.g., global constraints like *regular* or *bin-packing*) and some of them are supported by only a few solvers. Moreover, no standard input language for CSP exists and there are not exhaustive and immediately available benchmarks. These limitations somehow affect the available CSP (portfolio) solvers. The first and the only CSP portfolio solver that won an international competition (precisely, the ICSC 2008) was CPHydra [151]. The actual use of CPHydra is however limited, since it uses a restricted number of dated constituent solvers and it can not deal with optimization problems. Moreover, in [176] the authors point out that arguably the good results of CPHydra in ICSC 2008 could be attributed to the fact that the majority of the instances used in the

[1] "What suffices is enough".

ICSC were publicly available before the competition. A more recent CSP portfolio approach is Proteus [104]. It does not rely purely on CSP solvers, but may decide to encode a CSP instance into SAT. Similarly to CPHydra, Proteus is not able to solve COPs and still relies on the XCSP format. Conversely, given its recent introduction, it never attended an international competition.

Given this situation, the first objective of this thesis was to shed more light on CSP portfolio solvers. We started with a systematic study of different portfolio approaches for CSPs, with the aim of evaluating the many different techniques and solvers available. The first contribution —discussed in detail in Section 4.1— has therefore been an empirical evaluation of different CSP portfolio approaches by using the format, the dataset, and the solvers of the ICSCs 2008/2009. In particular, in such a work we compared by means of simulations some state-of-the-art SAT portfolios w.r.t. CPHydra and some simpler approaches built on top of some ML classifiers.

After this first evaluation, we realized that the situation was actually changing. The XCSP format used to model the CSPs in the ICSC has been in practice replaced by MiniZinc [146]. New solvers supporting MiniZinc have been developed and tested and the only active competition for evaluating CP solvers is nowadays the MiniZinc Challenge. We therefore extended the research described in Section 4.1 by performing a more extensive evaluation. We have enhanced our previous work by evaluating more and more recent solvers, by fully supporting the MiniZinc language (while still retaining compatibility with XCSP), and by using a new extensive dataset of CSP instances. Furthermore we developed SUNNY, a lazy CSP portfolio algorithm. In Section 4.2 we provide the details of this second evaluation.

4.1 A First Empirical Evaluation

In [7] we tried to perform a first step towards the clarification of the importance of portfolio approaches for solving CSPs. We investigated to what extent a portfolio approach can increase the performances of a single CSP solver and which could be

the best portfolio approaches, among the several existing, for solving CSPs. We basically implemented two classes of CSP portfolio solvers, building portfolios of up to 16 constituent solvers. In the first class we used relatively simple, off-the-shelf ML classification algorithms in order to define the solver selectors. In the second class we instead tried to adapt the best practice in SAT portfolio solving to the CSP setting. In addition, we considered the aforementioned CPHydra solver.

It is worth noticing that adapting portfolios techniques from other fields is not trivial: in most cases the lack of documentation and usability of a portfolio solver has forced the re-implementation of (part of) the approach. Although we tried to reproduce these approaches as faithfully as possible, they are not (and cannot be) the original ones. For the sake of simplicity and readability, in the following we refer to them with their original names (e.g., SATzilla or 3S) instead of using other notational conventions (e.g., SATzilla-like or 3S-like). The rest of this section provides a detailed description of the experiments conducted.

4.1.1 Methodology

In this section we explain the methodology and the main components used to conduct our experiments. All the approaches have been validated by using Intel®Dual-Core 2.93GHz computers with 2 GB of RAM and Ubuntu 12.04 Operating System. The code developed to conduct the experiments is available at `http://www.cs.unibo.it/~amadini/cpaior_2013.zip`.

4.1.1.1 Dataset

We tried to perform our experiments on a set of instances as realistic and large as possible. We gathered the publicly available benchmarks of ICSC 2008, consisting of CSPs already encoded in a normalized XCSP format.[2] In addition, we added to the dataset a number CSP instances coming from the MiniZinc 1.6 benchmarks. In order to use these instances, we first flattened the original MiniZinc models into

[2] The ICSC 2009 did not introduce new instances w.r.t. the ICSC 2008.

FlatZinc (by using the standard converter `mzn2fzn` [145]) and we then compiled the flattened models into XCSP (by exploiting the `fzn2xcsp` converter provided by the MiniZinc suite). Unfortunately, since FlatZinc is more expressive than XCSP, we could not convert all the models (e.g., the optimization problems were discarded).

The final dataset was built by considering 7163 instances taken from the ICSC and 2419 instances collected from the MiniZinc benchmarks. Moreover, we discarded all the instances solved by Mistral during the first 2 seconds computation of the dynamic features (see Section 4.1.1.3 for more details). We thus obtained a final dataset $\mathbb{D}$ containing 4547 instances encoded in XCSP language (3554 from the ICSC and 993 from MiniZinc benchmarks).

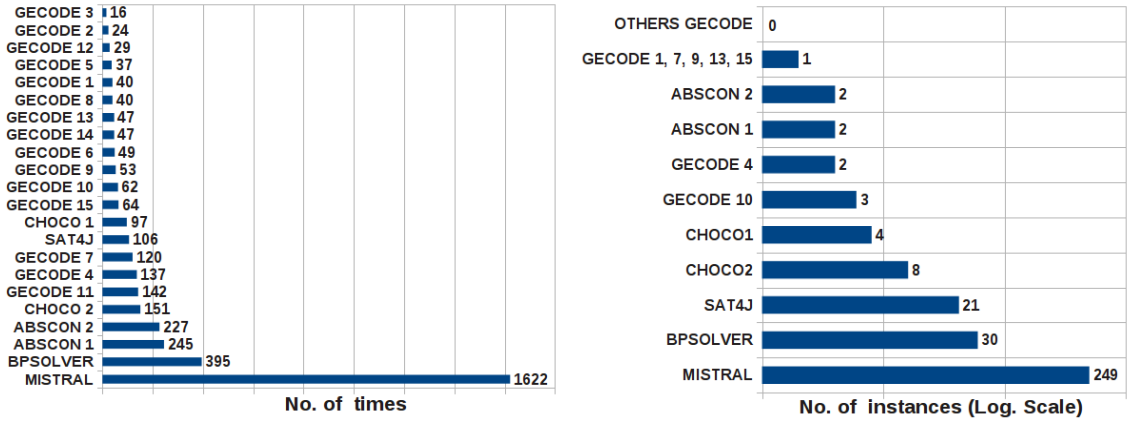
4.1.1.2 Solvers

We decided to build our portfolios by using some of the best ranked solvers of the ICSC, namely AbsCon (2 versions), BPSolver, Choco (2 versions), Mistral and Sat4j. Moreover, by exploiting a specific plug-in described in [141], we were able to use also 15 different versions of Gecode constraint solver.[3] The set $\mathbb{S}$ of the available constituent solvers was therefore constituted by 22 different solvers, all capable of processing CSP instances defined in the XCSP format.

In order to perform the experiments, we run every solver of $\mathbb{S}$ on each problem of the dataset $\mathbb{D}$ by using a solving time limit of $T = 1800$ seconds (the same used in the ICSC). In total, we therefore evaluated $|\mathbb{S} \times \mathbb{D}| = 22 \cdot 4547 = 100034$ runs. Note that 797 instances of $\mathbb{D}$ could not be solved by any solver of $\mathbb{S}$ within T seconds. Despite most portfolio approaches tend to discard such instances from the dataset, we instead decided to keep them. This is because we did not want to get rid of a body of knowledge that could make a contribution in terms of solver selection (e.g., the SUNNY algorithm that introduced in Section 4.2.1 uses a backup solver for covering the unsolved instances).

[3] Indeed, Gecode didn't attend the competition. The different versions of Gecode have been obtained by tuning the search parameters and the variable selection criteria.

Figure 4.1a indicates the relative speed of the different solvers by showing, for each solver, the number of instances a solver is the fastest one. As it can be seen, Mistral is by far the best solver, since it is faster than the others for 1622 instances (36% of $\mathbb{D}$). In Figure 4.1b, following [187], we show instead a measure of the marginal contribution of each solver, i.e., how many times a solver is able to solve instances that no other solver can solve. Even in this case Mistral is by far the best solver, almost one order of magnitude better than the second one. Note that there are also eight versions of Gecode that do not give any contribution in terms of solved instances.



(a) No. of times a solver is faster. (b) Marginal contributions.

Figure 4.1: Solvers statistics.

4.1.1.3 Features

In order to train and test the portfolio approaches, we extrapolated a set of 44 features from each instance of $\mathbb{D}$. We essentially used the 36 features of CPHydra [151] plus some additional features derived from the *Variable Graph* (VG) and the *Variable-Constraint Graph* (VCG) of the XCSP instances. Whilst the majority of these features are static, some of them are computed by collecting data from short runs of Mistral. Among the syntactical features we can mention the number of variables, the number of (global) constraints, the number of constants, the size of

the domains and the *arity* of the predicates. The dynamic features instead take into account the number of nodes explored and the number of propagations performed by Mistral within a time limit of 2 seconds. Table 4.1 briefly summarizes the features extracted; for a more detailed description we refer the reader to [116] since we used them off-the-shelf and they are not a contribution of this thesis.

Dynamic features (1–4): log average variables weight, log nodes, log propagation, log standard deviation, variables weight;
Logarithmic features (5–16): bits, boolean, constants, constraints, extra bits, extra boolean, extra ranges, extra values, lists, ranges, search variables, values;
General purpose features (17–18): max arity, all-different constraints;
Percent features (19–28): all-different, average continuity, cumulative, decompose, element, extensional, gac predicate, global, min continuity, weighted sum;
Perten features (29–34): average predicate arity, average predicate shape, average predicate size, binary ext, large ext, naryext;
Square root features (35–36): average domain size, max domain size;
Additional features (37–44): ratio, reciprocal ratio, log VG average, log VG standard deviation, log VCG average constraint, log VCG average variable, log VCG standard deviation constraint, log VCG standard deviation variable.

Table 4.1: Features extracted by CPHydra (1–36) plus additional features (37–44).

As mentioned in 4.1.1.1, $\mathbb{D}$ was obtained by discarding all the problems that Mistral solved within this 2 seconds. This is because in this case no solver selection is needed: the instance is already solved during the features extraction. The time needed to compute these features was often negligible: the average feature computation time was 2.47 seconds with a standard deviation of 3.54 and a maximum of 93.1 seconds.

4.1.1.4 Validation

In order to evaluate and compare the different portfolio solvers we used a 5-repeated 5-fold cross-validation (see Section 3.1 on page 31, and [15]). The dataset $\mathbb{D}$ was

randomly partitioned in 5 disjoint folds $\mathbb{D}_1, \mathbb{D}_2, \ldots, \mathbb{D}_5$ by using in turn a fold $\mathbb{D}_i$ as the test set and the union $\bigcup_{j \neq i} \mathbb{D}_j$ of the remaining folds as the training set. To avoid overfitting problems the random generation was repeated 5 times, thus obtaining 25 different test sets. Every test set was therefore constituted by approximately 909 instances, while every training set consisted of approximately 3638 instances.

For every instance of every test set we computed the solving strategy proposed by a particular portfolio approach. Then, we simulated it by exploiting the already computed runtimes, checking if the solving strategy was able to solve the instance within $T = 1800$ seconds. To evaluate the performances of a (portfolio) solver we used two metrics: the *Percentage of Solved Instances* (PSI) and the *Average Solving Time* (AST). The PSI metric measures the percentage of instances solved by a particular solver within the timeout T, while the AST refers to the average time needed by a solver to solve an instance. Formally, we can define such metrics as follows:

Definition 4.1 (`proven`, PSI) *Let us fix a dataset $\mathbb{D}$ of CSPs, a set $\mathbb{S}$ of CSP solvers, and a solving timeout T. Given a problem $p \in \mathbb{D}$ and a solver $s \in \mathbb{S}$, we define the binary function* $\texttt{proven} : \mathbb{S} \times \mathbb{D} \to \{0, 1\}$ *as:*

$$\texttt{proven}(s, p) := \begin{cases} 1 & \text{if solver } s \text{ solves the problem } p \text{ in less than } T \text{ seconds;} \\ 0 & \text{otherwise} \end{cases}$$

Given a set of problems $P \subseteq \mathbb{D}$, the PSI of solver s on P is given by:

$$\mathrm{PSI}(s, P) := \frac{100}{|P|} \sum_{p \in P} \texttt{proven}(s, p)$$

Definition 4.2 (`time`, AST) *Let us fix a dataset $\mathbb{D}$ of CSPs, a set $\mathbb{S}$ of CSP solvers, and a solving timeout T. Given a problem $p \in \mathbb{D}$ and a solver $s \in \mathbb{S}$,*

we define the function $\texttt{time} : \mathbb{S} \times \mathbb{D} \to [0, T]$ *as:*[4]

$$\texttt{time}(s, p) := \begin{cases} t & \text{if solver } s \text{ solves the problem } p \text{ in } t < T \text{ seconds;} \\ T & \text{otherwise.} \end{cases}$$

Given a set of problems $P \subseteq \mathbb{D}$*, the AST of solver* s *on* P *is given by:*

$$\mathrm{AST}(s, P) := \frac{1}{|P|} \sum_{p \in P} \texttt{time}(s, p)$$

In the following, where not ambiguous, for the sake of readability we will omit the arguments (s, P) of PSI and AST metrics.

Table 4.2 shows the PSI and AST performance of the solvers of $\mathbb{S}$ on the whole dataset $\mathbb{D}$. Note that —apart from Mistral which is firmly at the top— there are some differences w.r.t the ranks reported in Figure 4.1a and 4.1b on page 45. As an example, look at the performance of Sat4j: despite it has a good marginal contribution (it is 9$^{\text{th}}$ in 4.1a and 3$^{\text{rd}}$ in 4.1b) it is just 20$^{\text{th}}$ if we consider the overall PSI.

Solver	**PSI**	**AST**	**Solver**	**PSI**	**AST**
Mistral	63.95	724.30	Gecode13	36.35	1237.98
Choco2	54.17	899.43	Gecode6	30.57	1317.17
Choco1	53.57	908.20	Gecode11	28.59	1352.61
AbsCon2	47.53	1005.96	Gecode7	28.44	1355.76
AbsCon1	47.09	1012.61	Gecode1	27.14	1376.28
BPSolver	39.92	1193.86	Gecode12	24.17	1411.22
Gecode4	36.82	1230.86	Gecode8	24.17	1412.12
Gecode9	36.77	1232.47	Gecode2	23.36	1424.98
Gecode10	36.53	1231.01	Sat4j	20.26	1491.39
Gecode15	36.51	1235.69	Gecode5	11.39	1612.90
Gecode14	36.46	1236.27	Gecode3	11.35	1617.04

Table 4.2: Solvers performances.

[4]Note that the `time` metric is also called *Penalized Average Runtime* (PAR). Sometimes (e.g., in SAT competitions) PAR is replaced by PAR10, which penalizes the absence of answer with $10 \cdot T$ seconds instead of T seconds.

4.1.1.5 Portfolios composition

We pre-computed and fixed 15 portfolios $\Pi_m \subseteq \mathbb{S}$ of different size $m = 2, 3, \ldots, 16$. Each portfolio Π_m was computed by a local search algorithm aimed at maximizing the potential PSI of Π_m, breaking the ties by minimum AST.[5] Note that we did not use all the 22 solvers since we realized that for $m > 16$ all the portfolios potentially solve the same number of instances (i.e., adding a new solver potentially reduces the AST, but not increases the PSI).

The portfolios composition by varying the portfolio size is summarized in Table 4.3. Note that such a composition not necessarily reflects the ranking showed in Figure 4.1a, 4.1b and Table 4.2. As an example, $\Pi_2 = \{\text{Mistral}, \text{AbsCon2}\}$ allows to solve 3206 instances, while $\{\text{Mistral}, \text{BPSolver}\}$ and $\{\text{Mistral}, \text{Choco2}\}$ allow to solve 3186 and 3142 instances respectively. Since even in this case Mistral is the overall best solver, in the rest of the section it will be also called the *Single Best Solver* (SBS) of the portfolio(s).

4.1.1.6 Off-the-shelf approaches

A first class of algorithms that we used as solver selectors for our evaluation is represented by what we will call the *off-the-shelf* (OTS) approaches. OTS approaches are essentially ML classifiers that we use to predict the presumed best solver of the portfolio for a given test instance. For every $p \in \mathbb{D}$ we associated a label l_p corresponding to the solver able to solve such instance in less time. For all the instances not solvable by any solver of the portfolio we used a special label `NO-SOLVER`. The solver designated to solve a given instance is the one predicted by the classifier. In the cases where the solver predicted is `NO-SOLVER`, a pre-designated *backup solver* is

[5] Precisely, each Π_m was computed by using a hill-climbing approach that, starting from an initial random-generated portfolio Π_m^0, for $n \geq 1$ iteratively tries to find a portfolio Π_m^n of size m that solves more instances (or solves the same number of instances more quickly) than Π_m^{n-1}. The transition from Π_m^{n-1} to Π_m^n is obtained by replacing a random subset $S_{n-1} \subseteq \Pi_m^{n-1}$ with a subset $S_n \subseteq \mathbb{S}$ so that $\Pi_m^n = (\Pi_m^{n-1} \setminus S_{n-1}) \cup S_n$ and $|\Pi_m^n| = m$.

No. Solvers	Portfolio Composition							
2	AbsCon2	Mistral						
3	Gecode15	AbsCon2	Mistral					
4	Gecode15	AbsCon2	Choco2	Mistral				
5	Gecode15	AbsCon2	Choco2	BPSolver	Mistral			
6	BPSolver	Gecode15	Mistral	AbsCon2	Choco2	Sat4j		
7	BPSolver	Gecode15	Gecode10	Mistral	AbsCon2	Choco2	Sat4j	
8	BPSolver	Gecode15	Gecode7	Gecode10	Mistral	AbsCon1	Choco2	Sat4j
9	BPSolver	Gecode15	Gecode7	Gecode10	Sat4j	AbsCon1	Choco1	Choco2
	Mistral							
10	BPSolver	Gecode15	Gecode7	Gecode10	Gecode13	Sat4j	AbsCon1	Choco1
	Choco2	Mistral						
11	BPSolver	Gecode15	Gecode7	Gecode10	Gecode13	Mistral	AbsCon2	AbsCon1
	Choco1	Choco2	Sat4j					
12	Gecode12	BPSolver	Gecode15	Gecode7	Gecode10	Gecode13	Sat4j	AbsCon2
	AbsCon1	Choco1	Choco2	Mistral				
13	Gecode9	Gecode12	BPSolver	Gecode4	Gecode7	Gecode10	Gecode13	Mistral
	AbsCon2	AbsCon1	Choco1	Choco2	Sat4j			
14	Gecode9	Gecode12	BPSolver	Gecode4	Gecode7	Gecode1	Gecode10	Gecode13
	Sat4j	AbsCon2	AbsCon1	Choco1	Choco2	Mistral		
15	Gecode9	Gecode12	BPSolver	Gecode15	Gecode4	Gecode7	Gecode1	Gecode10
	Gecode13	Sat4j	AbsCon2	AbsCon1	Choco1	Choco2	Mistral	
16	Gecode9	Gecode12	BPSolver	Gecode15	Gecode14	Gecode7	Gecode1	Gecode10
	Gecode13	Sat4j	AbsCon2	AbsCon1	Gecode4	Choco1	Choco2	Mistral

Table 4.3: Portfolios composition.

instead selected.[6] To train and test the models we used the WEKA tool [91] which implements some of the most well-known and widely used classification algorithms. In particular, we used the following classifiers:

- *IBk* (`weka.classifiers.lazy.IBk`), a k-Nearest Neighbours classifier;
- *DecisionStump* (`weka.classifiers.trees.DecisionStump`), based on a decision stump. Usually used in conjunction with a boosting algorithm;
- *J48* (`weka.classifiers.trees.J48`), uses (un)pruned C4.5 decision trees;

[6] A backup solver is a special solver of the portfolio aimed to handle exceptional circumstances (e.g., premature failures of other solvers). We chose Mistral as backup solver since it is the SBS of each considered portfolio.

- *NaiveBayes* (`weka.classifiers.bayes.NaiveBayes`), relies on a Naive Bayes classifier using estimator classes;
- *OneR* (`weka.classifiers.rules.OneR`), uses the minimum-error attribute for prediction, discretising numeric attributes;
- *PART* (`weka.classifiers.rules.PART`), exploits PART decision lists;
- *Random Forest* (RF) (`weka.classifiers.trees.RandomForest`), constructs forests of random trees;
- *SMO* (`weka.classifiers.functions.SMO`), implements a sequential minimal optimization algorithm for training a support vector classifier.

In order to boost the prediction accuracy, we have also used the AdaBoostM1 e LogitBoost *meta-classifiers* and we performed a *parameter tuning* of the classifiers configurations.[7] The latter task was performed by following the best practices — when they were available— or by manually trying different parameters starting from the default WEKA settings. For instance, for the SMO classifier we used a Radial Basis Function kernel and we performed a grid search over the C and γ parameters [183].

4.1.1.7 Other approaches

The OTS approaches have been compared against CPHydra and some state-of-the-art SAT portfolio solvers.

CPHydra [151] is the only CSP portfolio solver that won an international competition.[8] This solver uses a k-NN algorithm in order to compute a schedule of the constituent solvers that maximizes the number of instances solved in the neighbourhood. A weak point of CPHydra is that it is not very scalable w.r.t. the number of

[7] We have also tried to apply *oversampling* techniques (Synthetic Minority Over-sampling Technique [41] to be precise). This however was found to be ineffective.

[8] At the time when the experiments were conducted, CPHydra was actually the only available CSP portfolio solver.

the constituent solvers: as pointed out by its authors, "for a large number of solvers a more sophisticated approach would be necessary". It does not employ heuristics for sorting the scheduled solvers and, consequently, for minimizing the solving time. Nevertheless, by using a rather small size portfolio (3 solvers) CPHydra was able to win the ICSC 2008.

In order to reproduce the CPHydra approach, we computed the solvers schedule by exploiting the code available at `http://homepages.laas.fr/ehebrard/cphydra.html`. Since this approach does not scale very well w.r.t. the size of the portfolio we were able to simulate CPHydra only for portfolios consisting of at most eight solvers. Note that for computing the PSI and the AST of CPHydra we did not take into account the time it needed to compute the solvers schedule. Thus, the results of CPHydra presented in Section 4.1.2 can be considered as an upper bound of its real performances.

3S (SAT Solver Selector) [113] is a SAT portfolio solver that conjugates a fixed-time static schedule with the dynamic selection of one long-running solver. Exploiting the fact that a lot of SAT instances are extremely easy for one solver and almost impossible to solve for others, 3S first executes for 10% of the time limit short runs of its constituent solvers. The schedule of solvers, obtained by solving an optimization problem similar to the one tackled by CPHydra, is computed offline (i.e., during the learning phase on training data). Then, at run time, if a given instance is still not solved a designated solver is executed for the remaining time. This solver is chosen among the ones that are able to solve the majority of the most k-similar instances in the training set. 3S solves the scalability issues of CPHydra because the schedule is computed offline. Moreover, it also uses other techniques for improving the performance of the algorithm selector. This allowed 3S to use a portfolio of 21 solvers and to be the best-performing dynamic portfolio in the International SAT Competition 2011.

For computing the static schedule of 3S approach we did not use the original code since it is not publicly available. We instead used the MIP solver *Gurobi* [89] for solving the linear program described in [113]. In order to reduce the search space,

we imposed an additional constraint requiring every solver to run for a discrete number of seconds. In this way it was possible to get an optimal schedule of solvers in reasonable time.[9] If a problem is not solved by the static schedule in the first $T/10 = 180$ seconds, we select for the remaining time the constituent solver that solves the most instances in the k-NN neighbourhood. In order to comply with CPHydra approach, we set $k = 10$.

ISAC (Instance-Specific Algorithm Configuration) is a tool aimed at optimally tuning the configuration of an highly-parametrised solver. ISAC statically clusters the training set by using the g-means algorithm [93] and identifies the best parameters setting for each cluster by means of GGA algorithm [13]. When a new instance needs to be classified, ISAC determines the cluster with the nearest center and selects the precomputed parameters for such cluster.

Thanks to the code kindly provided by Yuri Malitsky, we were able reproduce the "*Pure Solver Portfolio*" approach described in [134] for SAT problems. We first clustered the training instances and then mapped every cluster to the corresponding best solver, i.e., the solver that solves more instances in the cluster. For every test instance we determined the closest cluster according to the Euclidean distance and run the corresponding best solver.

SATzilla is a SAT portfolio solver that relies on runtime prediction models to select the solver that (hopefully) has the fastest running time on a given problem instance. In the International SAT Competition 2009, SATzilla won all the three major tracks. More recently a new powerful version of SATzilla has been proposed [190]. Instead of using regression-based runtime predictions, the newer version uses a weighted random forest approach provided with an explicit cost-sensitive loss function punishing misclassifications in direct proportion to their impact on portfolio performance. This last version consistently outperformed the previous versions of SATzilla and the other competitors in the SAT Challenge 2012.

We simulated the SATzilla approach by developing a MATLAB implementation

[9] Note that 3S uses column generation techniques to reduce the size of the problem, in spite of the solution optimality.

of the cost-sensitive classification model described in [190] with the only exception that ties during solvers comparison are broken by selecting the solver that in general solves the largest number of instances. We did this for simplicity: even if the code of SATzilla is publicly available, it would have been very difficult to adapt it for our purposes.

4.1.2 Results

We remark that in order to evaluate the performance we simulated the execution of the solvers considering the already computed solving times. We therefore assume that all the solvers have a deterministic behaviour. Moreover, in order to present a more realistic scenario, the solving time of each portfolio approach also includes the features extraction time.

In the rest of the section we present and discuss the experimental results in terms of PSI and AST, by setting the solving timeout to $T = 1800$ seconds.

4.1.2.1 Percentage of Solved Instances

Figure 4.2 depicts the PSI performance of OTS approaches only, eventually boosted by a meta classifier whenever its use improved the performance. As additional baselines, we introduced the performance of Mistral (SBS) and of the *Virtual Best Solver* (VBS), i.e., an oracle that for every instance always chooses the best solver according to a given metric. Formally, we can define the Virtual Best Solver as follows:

Definition 4.3 (VBS) *Let us fix a dataset $\mathbb{D}$ of problems, a set $\mathbb{S}$ of solvers, and a* performance metric *$f : \mathbb{S} \times \mathbb{P} \to \mathbb{R}$ to be maximized. We define the* Virtual Best Solver *(VBS) of $\mathbb{S}$ (according to f) as a fictitious portfolio solver $VBS_f(\mathbb{D}, \mathbb{S})$ that, for every $p \in \mathbb{D}$, always selects the solver $s \in \mathbb{S}$ that maximizes $f(s, p)$.*

Note that the definition of Virtual Best Solver depends only on the dataset $\mathbb{D}$, the solvers $\mathbb{S}$, and the performance metric f. In particular, $\mathbb{D}$ may contain both

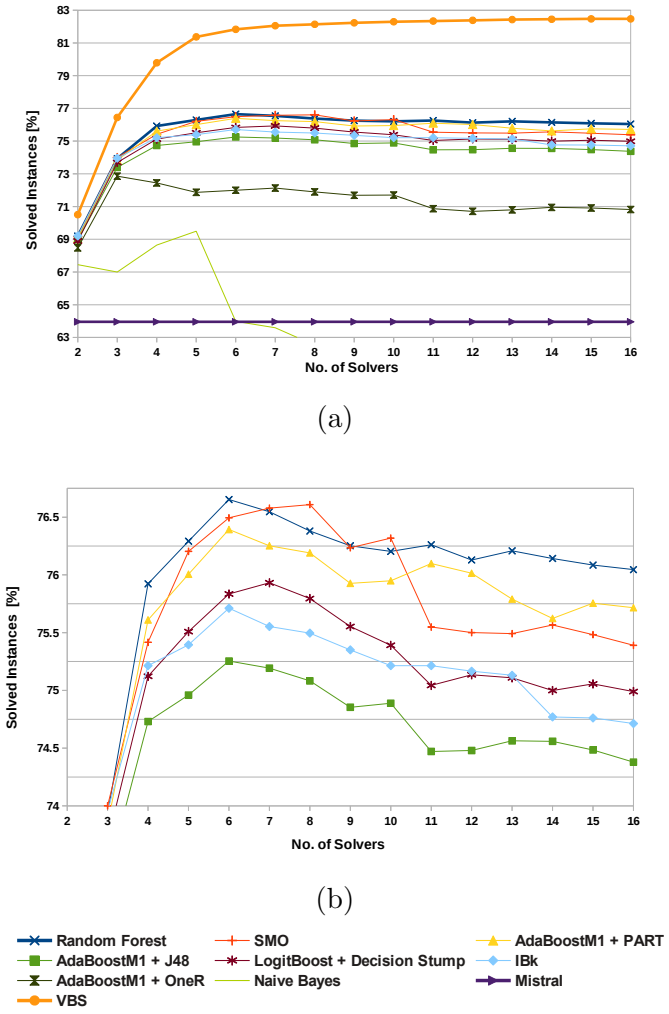


Figure 4.2: Percentage of Solved Instances of OTS approaches.

CSPs and COPs. For ease of reading, from now on we will simply use the notation VBS in place of $\text{VBS}_f(\mathbb{D}, \mathbb{S})$.

Figure 4.2b shows more in detail the best OTS approaches of Figure 4.2a.

From Figure 4.2a we can see that almost all the approaches greatly outperform Mistral, which has a PSI of 63.95%. The only exception is the Naive Bayes classifier, that with more than six solvers is even worse than Mistral. The VBS solves 82.47% of the instances with a portfolio of ≥ 14 solvers, while the best OTS approach is Random Forest (RF) that reaches a PSI of 76.65% with a portfolio of six solvers.

However, Figure 4.2b shows that some other approaches are pretty close to Random Forest: for example, SMO has a peak performance of 76.61% with a portfolio of eight solvers.

According to the *Student's t-test* [97], we have also analysed the statistical significance of the results. We considered statistically significant two samples of data[10] having a p-value less or equal than 0.05 (i.e., the probability that the observed difference is due to chance should be less than 5%). We realized that —by fixing the portfolio size— for portfolios of size between 5 and 10 the performances of RF are not statistically significant w.r.t. the corresponding SMO performances. Conversely, RF is statistically better when compared to all other approaches. This confirms the similarity of the performance between RF and SMO classifiers, as indeed can be seen from the plots.

As far as the portfolio size is concerned, we noticed that for every classifier the prediction becomes inaccurate after a given size. Indeed, despite the use of a larger portfolio means that potentially more instances could be solved, we note that the best performance has been obtained by using portfolios from 6 to 8 solvers. For some classifiers the performance deterioration is quite significant: e.g., by using 16 solvers the SMO classifier solves 1.22% instances less (i.e, $1.22 \cdot 4547 \cdot 5 \,/\, 100 \approx 277$ problems less) than using 8 solvers. We also compared the statistical significance by fixing an approach and varying the portfolio size. It turned out that for RF the difference between the peak performance (obtained with 6 solvers) and the performances it reached with $k \neq 6$ solvers was not statically significant only for $k = 7$. Similarly, for SMO approach the only not statically significant difference is between 7 and 8 solvers (peak performance). These results confirm the intuition that it is sometimes better to reasonably limit the portfolio size.

In Figure 4.3 we compare the performance of RF (i.e., the best OTS approach) with those of the aforementioned 3S, CPHydra, ISAC, and SATzilla approaches.

[10] We define the data sample for a given portfolio solver as a binary vector $(x_1, x_2, \ldots, x_n) \in \{0,1\}^n$ corresponding to the $n = 4547 \cdot 5 = 22735$ tested instances. For $i = 1, 2, \ldots, n$, each x_i is set to 1 if and only if the portfolio solver is able to solve the instance i within the time limit T.

For ease of reading, we included the VBS but not the SBS. As already stated, due to the computational cost of computing the schedule of solvers, for CPHydra the results are limited to a maximum of 8 solvers.

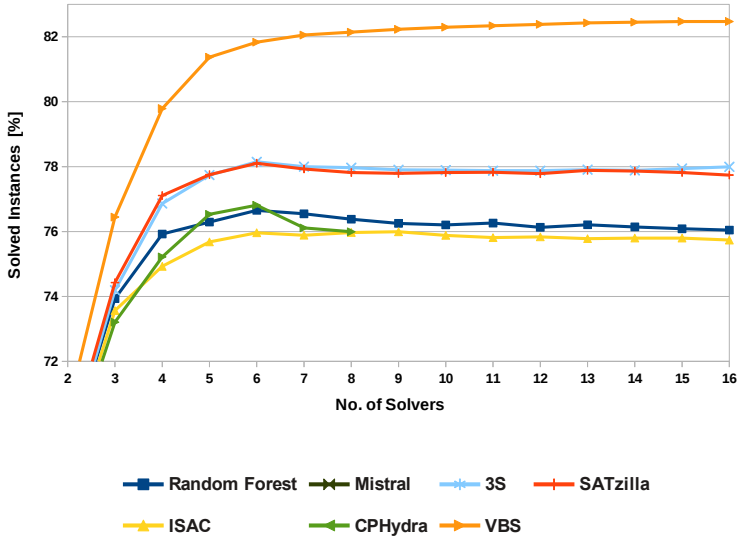


Figure 4.3: PSI of the best approaches.

In this case it is possible to notice that the best SAT portfolio approaches, namely 3S and SATzilla, achieve the best results. 3S is able to solve slightly more instances than SATzilla (3S has a peak PSI of 78.15% against the 78.10% peak performance of SATzilla). Curiously enough, even though conceptually 3S and SATzilla use very different approaches, they have very close performances. This is confirmed also from a statistical point of view: the performance difference is not significant when using the same portfolio. 3S and SATzilla are instead statistically better than all the other approaches when using more than 3 solvers (e.g., 3S is able to close 26% of the gap between Random Forest and the VBS). Moreover, their decay of performances is less pronounced when increasing the portfolio size. As for the OTS approaches, the best performance was reached with a relatively small portfolio (6 solvers). In particular, the peak performances of both 3S and SATzilla are statistically significant w.r.t. their performances with different portfolio size.

CPHydra approach achieves good performance (the best is 76.81% with 6 solvers) slightly overcoming RF and SMO. However, for 7 and 8 solvers there is a worsening and for more than 8 solvers this approach becomes intractable.

The performances of ISAC approach are instead slightly worse than those of RF and SMO. Indeed, ISAC reaches a maximum PSI of 75.99% (9 solvers).

4.1.2.2 Average Solving Time

Figure 4.4 shows the AST of the best approaches reported in Figure 4.3, including also the performance of Mistral.

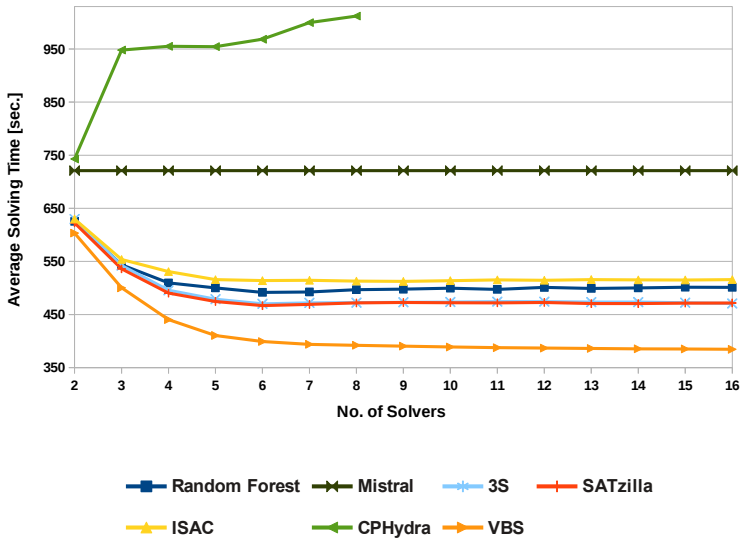


Figure 4.4: AST of the best approaches.

There are basically two things that draw the attention in the plot. On the one hand, for all the approaches except CPHydra one can observe a clear anti-correlation between the PSI and the AST. In other terms, by comparing Figure 4.3 and 4.4, it can be noted that the AST of such approaches decreases proportionally to the increase of the PSI. This strong anti-correlation is also confirmed by the *Pearson coefficient* which is always below -0.985. The substantial break even between SATzilla

and 3S is also confirmed by the fact that SATzilla is on average slightly faster than 3S (while, as seen above, 3S solves a few problems more). In particular the AST of SATzilla is 466.82 seconds (6 solvers), against the 470.30 seconds of 3S. The VBS instead reaches a minimum AST of 384.46 seconds with 16 solvers.

On the other hand, CPHydra behaves differently. As previously stated, this approach was not developed for minimizing the AST and therefore it does not employ any heuristic for deciding the ordering of the scheduled solvers. As a consequence, CPHydra turns out to be the only portfolio solver that not minimizes the AST even when the PSI increases. Figure 4.4 shows that the AST of CPHydra is always worse than those of Mistral (721.03 seconds) for each portfolio size, even if its PSI is always higher. It is therefore not surprising that CPHydra is the only approach for which PSI and AST have a positive correlation: the Pearson coefficient between its PSI and AST values is 0.921, denoting an almost linear relationship.

4.1.2.3 Summary

In the empirical evaluation presented in this section we have implemented and compared different CSP portfolio solvers. The experimental results we got have confirmed the potential effectiveness of portfolio approaches also in CSP field. Indeed, the single best solver of the portfolio was almost always significantly outperformed.

It is not surprising that the best practices in the SAT field, namely SATzilla and 3S, are also reflected in the more general CSP setting. The considerable performances achieved by these approaches encouraged us to combine them, by exploiting the static solver schedule of 3S together with the dynamic selection of a long-running solver determined by SATzilla approach. Curiously, the performances of this combined approach did not improve the individual performances of 3S and SATzilla.

Another interesting fact is that, for almost all the considered portfolio approaches except CPHydra, there is a strong anti-correlation between the average solving time and the number of solved instances. Minimizing the average solving time in this setting can therefore lead to solve more instances and vice versa.

However, the OTS approaches are not that far from the peak performances and —due to their simplicity— might potentially be used in dynamic scenarios were the time to build the prediction model matters.

4.2 An Enhanced Evaluation

The extensive evaluation reported in Section 4.1 was helpful to have a general idea of the potential benefits that portfolios might bring to CSP field. However, since then the situation has evolved.

Firstly, the XCSP format used to model the CSPs in the ICSC has been actually replaced by MiniZinc [146]. New solvers supporting MiniZinc have been developed and tested, and the only remaining constraint solvers competition is nowadays the MiniZinc Challenge [176]. A fundamental difference between XCSP and MiniZinc concerns the expressiveness: unlike XCSP, MiniZinc also supports optimization problems (which constitute about 75% of the MiniZinc Challenge problems).

Secondly, in the experiments described in Section 4.1 we reproduced and compared existing portfolio approaches, but we did not introduce any new portfolio technique.

A second contribution of this thesis is therefore a major extension of the research we initiated in [7]. We performed an enhanced evaluation which extends and improves our previous work for several reasons:

- we evaluate portfolios with more and more recent solvers;
- we fully support the MiniZinc language, while still retaining compatibility with XCSP format;
- we use a new extensive dataset of CSP instances;
- we add to the evaluation SUNNY, a lazy portfolio approach we developed, and `sunny-csp`, a prototype for a new generation CSP portfolio solver built on top of SUNNY algorithm.

In the rest of this section we explain in detail these contributions. In particular, in Section 4.2.1 we describe the SUNNY algorithm, in Section 4.2.2 the methodology used to conduct the new experiments, and in Section 4.2.3 the experimental results we obtained.

4.2.1 SUNNY: a Lazy Portfolio Approach

As pointed out also by [164], probably one of the main causes of portfolios under-utilization lies in the fact that state-of-the-art portfolio solvers usually require a complex off-line training phase, and they are not suitably structured to incrementally exploit new incoming information. For instance, SATzilla builds its prediction model by exploiting a weighted Random Forest approach; 3S computes an off-line schedule of solvers by exploiting column generation techniques; ISAC performs a clustering of the training instances.

Some approaches avoiding a heavy off-line training phase have been studied and evaluated [154, 151, 147, 72, 164]. These approaches are also referred as *lazy* [118]. Avoiding (or at least lightening) the training phase is advantageous in terms of simplicity and flexibility: it facilitates the treatment of new incoming problems, solvers and features. Unfortunately, among these lazy approaches only CPHydra was successful enough to win a solving competition (the ICSC 2008). In Section 4.1, however, we showed that non-lazy approaches derived from SATzilla and 3S performed better than CPHydra.

In this section we introduce SUNNY: a lazy portfolio approach for constraint solving. SUNNY exploits instances similarity to guess the best solver(s) to use. For a given problem p, SUNNY uses a k-Nearest Neighbours (k-NN) algorithm to select from a training set of known instances the subset $N(p, k)$ of the k instances closer to p. Then, it creates a schedule of solvers by considering the smallest sub-portfolio able to solve the maximum number of instances in the neighbourhood $N(p, k)$. The time allocated to each solver of the sub-portfolio is proportional to the number of instances it solves in $N(p, k)$.

The SUNNY algorithm is described below, while an extensive evaluation and comparison of its performance is reported in 4.2.3. Section 4.2.3.3 instead shows `sunny-csp`, an effective CSP portfolio solver that relies on SUNNY algorithm.

4.2.1.1 SUNNY Algorithm

One of the main empirical observations at the base of SUNNY is that usually combinatorial problems are extremely easy for some solvers and, at the same time, almost impossible to solve for others. Moreover, in case a solver is not able to solve an instance quickly, it is likely that such solver takes a huge amount of time to solve it. A first motivation behind SUNNY is therefore to select and schedule a subset of the constituent solvers instead of trying to predict the "best" one for a given unseen instance. This strategy hopefully allows one to solve the same amount of problems minimizing the risk of choosing the wrong solver.

Another interesting consideration is that using large portfolios not always lead to performance boost. In some cases the over-abundance of solvers hinders the effectiveness of the considered approach. As seen in Section 4.1, and pointed out also by [155], usually the best results are obtained by adopting a relatively small portfolio (e.g., ten or even less solvers).

Another motivation for SUNNY is that —as witnessed for instance by the good performance reached by CPHydra, ISAC, 3S, CSHC [131]— the "similarity assumption", stating that similar instances behave similarly, is often reasonable. It thus makes sense to use algorithms such as k-NN to exploit the closeness between different instances. As a side effect, this allows to relax the off-line training phase that, as previously stated, makes the majority of the portfolio approaches rarely used in practice.

Starting from these assumptions and driven by these motivations we developed SUNNY, whose name is the acronym of:

- *SUb-portfolio*: for a given instance, we select and run a suitable sub-portfolio (i.e., a subset of the constituent solvers of the portfolio);

- *Nearest Neighbours*: to determine the sub-portfolio we use a k-NN algorithm that extracts from the training set the k instances that are closer to the instance to be solved;

- *lazY*: the approach is lazy, since no explicit prediction model is built off-line.

In a nutshell, the underlying idea behind SUNNY is to minimize the probability of choosing the wrong solvers(s) by exploiting instance similarities in order to quickly get the smallest possible schedule of solvers.

The pseudo-code of SUNNY is presented in Listing 4.1. SUNNY takes as input the problem `inst` to be solved, the portfolio of solvers `solvers`, a backup solver `bkup_solver`, a parameter `k` (≥ 1) representing the neighbourhood size, a parameter `T` representing the solving timeout, and a knowledge base `KB` of known instances for each of which we assume to know the features and the runtimes for every solver of the portfolio.

When a new unseen instance `inst` comes, SUNNY first extracts from it a proper set of features via the function `getFeatures` (line 2). This function takes as input also the knowledge base `KB` since the extracted features need to be preprocessed in order to scale them in the range $[-1, 1]$ and to remove the constant ones. `getFeatures` returns the features vector `feat_vect` of the instance `inst`. In line 3, the function `getNearestNeigbour` is used to retrieve the `k` nearest instances `similar_insts` to the instance `inst` according to a certain distance metric (e.g., Euclidean). Then, in line 4 the function `getSubPortfolio` selects the minimum subset of the portfolio that allows to solve the greatest number of instances in the neighbourhood, by using the average solving time for tie-breaking. Formally, fixed a timeout T, a portfolio $\Pi = \{s_1, s_2, \ldots, s_m\}$, and a set of instances $P = \{p_1, p_2, \ldots, p_k\}$, we define an auxiliary function $\varphi : \mathcal{P}(\Pi) \to \{0, 1, \ldots, k\} \times \{0, 1, \ldots, m\} \times [0, T]$ such that, for each $S \subseteq \Pi$:

$$\varphi(S) := (k - \sum_{p \in P} \max\{\texttt{proven}(s, p) \mid s \in S\}, |S|, \sum_{s \in S, p \in P} \texttt{time}(s, p))$$

where $|S|$ is the cardinality of S and `proven`, `time` are defined as in Definitions 4.1 and 4.2 respectively. The function `getSubPortfolio` returns the subset of Π that minimizes w.r.t. the lexicographic ordering the set of triples $\{\varphi(S) \mid S \subseteq \Pi\}$.

```
1  SUNNY(inst, solvers, bkup_solver, k, T, KB):
2    feat_vect = getFeatures(inst, KB)
3    similar_insts = getNearestNeigbour(feat_vect, k, KB)
4    sub_portfolio = getSubPortfolio(similar_insts, solvers, KB)
5    slots = ∑_{s∈sub_portfolio} getMaxSolved({s}, similar_insts, KB, T) +
6          (k − getMaxSolved(sub_portfolio, similar_insts, KB, T))
7    time_slot = T / slots
8    tot_time = 0
9    schedule = {}
10   schedule[bkup_solver] = 0
11   for s in sub_portfolio:
12     solver_slots = getMaxSolved({s}, similar_insts, KB, T)
13     schedule[s] = solver_slots * time_slot
14     tot_time += solver_slots * time_slot
15   if tot_time < T:
16     schedule[bkup_solver] += T − tot_time
17   return sort(schedule, similar_insts, KB)
```

Listing 4.1: SUNNY Algorithm.

Once computed the sub-portfolio, we partition the time window $[0, \mathtt{T}]$ into `slots` equal time slots of size `T/slots`, where `slots` is the sum of the solved instances for each solver of the sub-portfolio plus the instances of `similar_insts` that can not be solved within the time limit `T`. In order to compute `slots`, we use the function `getMaxSolved`(S, `similar_insts`, `KB`, `T`) that returns the number of instances in `similar_insts` that the set of solvers S is able to solve within `T` seconds. In lines 9–10 the associative array `schedule`, used to define the solvers schedules, is initialized. In particular, `schedule[s] = t` if and only if a time window of t seconds is allocated to the solver s.

The loop enclosed between lines 11–14 assigns to each solver of the portfolio a number of time slots proportional to the number of instances that such solver can

solve in the neighbourhood. In lines 15–16 the remaining time slots, corresponding to the unsolved instances, are allocated to the backup solver. Finally, line 17 returns the final schedule, obtained by sorting the solvers by average solving time in `similar_insts`.

Example 4.1 *Let us suppose that* `solvers` $= \{s_1, s_2, s_3, s_4\}$, `bkup_solver` $= s_3$, `T` $= 1800$ *seconds,* `k` $= 5$, `similar_insts` $= \{p_1, ..., p_5\}$, *and the run-times of the problems* p_i *defined by* `KB` *as listed in Table 4.4.*

	p_1	p_2	p_3	p_4	p_5
s_1	T	T	**3**	T	**278**
s_2	T	**593**	T	T	T
s_3	T	T	**36**	**1452**	T
s_4	T	T	T	**122**	**60**

Table 4.4: Runtimes (in seconds). T indicates the timeout.

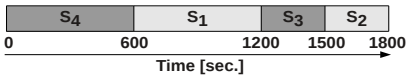


Table 4.5: Resulting schedule of the solvers.

The minimum size sub-portfolios that allow to solve the most instances (i.e., four instances) are $\{s_1, s_2, s_3\}$, $\{s_1, s_2, s_4\}$, *and* $\{s_2, s_3, s_4\}$.

SUNNY selects `sub_portfolio` $= \{s_1, s_2, s_4\}$ *since it has a lower average solving time (1270.4 sec., to be precise). Since* s_1 *and* s_4 *solve two instances,* s_2 *solves one instance and* p_1 *is not solved by any solver within* `T` *seconds, the time window* $[0, \texttt{T}]$ *is partitioned in* $2 + 2 + 1 + 1 = 6$ *slots: two assigned to* s_1 *and* s_4*, one slot to* s_2*, and one to the backup solver* s_3*. After sorting the solvers by average solving time in the neighbourhood we get the schedule illustrated in Table 4.5.*

Clearly, the proposed algorithm features a number of degrees of freedom. For instance, the underlying k-NN algorithm depends on the quality of the features extracted (and possibly filtered), on the choice of the neighbourhood size k, and on the distance metric adopted.

A potential weakness of SUNNY is that it could become impracticable for large portfolios. Indeed, in the worst case the complexity of `getSubPortfolio` is exponential w.r.t. the portfolio size since it has to evaluate $\varphi(S)$ for each subset $S \subseteq \Pi$. However, the computation of this function is almost instantaneous for portfolios containing up to 15 solvers and, as better detailed in Section 4.2.3.3, the sizes of the sub-portfolios are rather small for reasonable values of k.

Allocating the uncovered instances of $N(p, k)$ to the backup solver allows the assignment of some slots to the (hopefully) most reliable solver. This choice obviously biases the schedule toward the backup solver, but experimental results have proven the effectiveness of this approach.

Finally, we want to emphasize the differences between SUNNY and CPHydra. Although at first glance these approaches may seem pretty related (in fact, they are both lazy approaches that apply a k-NN algorithm to compute at runtime a schedule of the constituent solvers) there are some remarkable differences.

SUNNY puts the emphasis on finding a minimal subset of "good" solvers. The idea is to discard the "bad" ones, keeping only the solver(s) that give(s) a positive contribution in terms of solved instances. The schedule is then calculated on such subset of good solvers, without solving any Set Covering problem as done instead by CPHydra: only the number of instances solved in the neighbourhood matters. We can say that SUNNY relies on a "weak" assumption of similarity, i.e.: "similar instances are (not) solved by similar solvers". CPHydra instead computes a schedule that tries to maximize the number of solved instances in the neighbourhood. The assumption of CPHydra is stronger: "similar instances have similar runtimes".

Let us consider for instance the Example 4.1. The schedule returned by SUNNY is $[(s_4, 600), (s_1, 600), (s_3, 300), (s_2, 300)]$, while the one returned by CPHydra is $\{s_1\text{: } 716, s_3\text{: } 361, s_2\text{: } 600, s_4\text{: } 123\}$.[11] Note that we used a different notation for the two schedules since CPHydra does not use heuristics for sorting the solvers. Another

[11] This is the schedule returned by using the code available on-line and, in this case, by assuming a constant distance between the problem to be solved and those of the neighbourhood. Indeed, unlike SUNNY, CPHydra uses a weighted Euclidean distance for computing the schedule.

significant difference concerns the use of a backup solver. Unlike CPHydra, SUNNY allocates to the backup solver an amount of time proportional to the instances not solved in the neighbourhood. If, for example, the backup solver of Example 4.1 was s_1 instead of s_3, then the resulting schedule would be $[(s_1, 900), (s_4, 600), (s_2, 300)]$.

Finally, let us consider the time required for computing the solvers schedule. We empirically verified that CPHydra can take a long time (much more than 1800 seconds) for portfolios of size ≥ 8. Conversely, SUNNY computes instantaneously the solvers schedule for portfolios up to 12 solvers.

4.2.2 Methodology

Analogously to Section 4.1.1, in this section we explain the methodology used to conduct this new evaluation. In particular, we describe the new dataset of problems, the new solvers, and the new features we used. Even these experiments have been performed on Intel R Dual-Core 2.93GHz computers with 2 GB of RAM and Ubuntu 12.04 Operating System. The code developed is publicly available at `http://www.cs.unibo.it/~amadini/csp_portfolio.zip`.

4.2.2.1 Dataset

As already mentioned, in this evaluation we decided to switch from XCSP to MiniZinc language. However, to the best of our knowledge, the biggest existing dataset of CSP instances is still the one used in the ICSCs 2008/09. Since the CSPs of such dataset are encoded in XCSP, we developed a compiler from XCSP to MiniZinc called `xcsp2mzn`. Exploiting the fact that MiniZinc is more expressive than XCSP (i.e., the majority of the primitive constraints of XCSP are also primitive constraints of MiniZinc) the translation was pretty straightforward. The only notable difference concerns the compilation of extensional constraints, i.e., relations explicitly expressed in terms of the (not) allowed tuples. This kind of constraints are a native feature in XCSP only. In this case, we used the *table* global constraint

for encoding the allowed set of tuples, and a conjunction of disjunctions of inequalities for mapping the forbidden set of tuples.

The `xcsp2mzn` compiler allowed us to combine both the XCSP instances of ICSC and those coming from MiniZinc 1.6 suite and MiniZinc Challenge 2012. We then retrieved an initial dataset of 8600 instances, that we subsequently narrowed by properly discarding the "too easy" or "too complex" instances (see Section 4.2.2.3 for more details). We thus obtained a final dataset $\mathbb{D}$ of 4642 MiniZinc models.[12]

4.2.2.2 Solvers

In Section 4.1.1.2, we evaluated 22 versions of 6 solvers: AbsCon (2 versions), BProlog, Choco (2 versions), Mistral, Sat4j, Gecode (15 versions). In this evaluation we wanted instead to increase the number of solvers, avoiding the use of different versions of the same solver. This was done for not biasing the evaluation toward one or more solvers.

We considered a set $\mathbb{S}$ of eleven different solvers that attended the MiniZinc Challenge 2012, namely: BProlog, Fzn2smt, CPX, G12/FD, G12/LazyFD, G12/CBC, Gecode, iZplus, MinisatID, Mistral, and OR-Tools. To avoid misalignments, we used all of them with their default parameters, their global constraint redefinitions when available, and keeping track of their performances on every instance of $\mathbb{D}$ within a timeout of $T = 1800$ seconds.[13] In total, we therefore evaluated $|\mathbb{D} \times \mathbb{S}| = 4642 \cdot 11 = 51062$ runs.

Similarly to what done in Section 4.1.1.5 we built portfolios of different size $m = 2, 3, \ldots, 11$ by considering the subset $\Pi_m \subseteq \mathbb{S}$ with cardinality m which maximized the number of potential solved instances (possible ties were broken by minimizing the average solving time). A summary of the portfolios composition is reported in

[12] For the sake of uniformity and to ease the reading, we use the same notational conventions of Section 4.1. Obviously, this $\mathbb{D}$ is not the same dataset of Section 4.1.1.3, that consisted of 4547 XCSP instances.

[13] We decided to keep the timeout of the ICSC since the one of MiniZinc Challenge is lower (900 seconds). Note also that the versions of BProlog, Gecode, and Mistral considered here are not the same considered in the first evaluation described in Section 4.1.

Table 4.6. As can be noted, in the best case it is possible to solve at most 3698 instances. Even in this case, we kept in the dataset the 944 problems not solvable by any solver.

Size	Solvers	No. of Solved Inst.
2	MinisatID, Fzn2smt	3213
3	MinisatID, Fzn2smt, OR-Tools	3549
4	MinisatID, Fzn2smt, OR-Tools, iZplus	3606
5	MinisatID, CPX, Fzn2smt, OR-Tools, iZplus	3652
6	MinisatID, CPX, Fzn2smt, OR-Tools, iZplus, G12/CBC	3672
7	MinisatID, CPX, Fzn2smt, OR-Tools, G12/LazyFD, iZplus, G12/CBC	3684
8	MinisatID, CPX, Fzn2smt, OR-Tools, G12/LazyFD, Gecode, iZplus, G12/CBC	3692
9	MinisatID, CPX, Fzn2smt, OR-Tools, G12/LazyFD, Gecode, iZplus, Mistral, G12/CBC	3695
10	MinisatID, CPX, Fzn2smt, OR-Tools, G12/LazyFD, Gecode, iZplus, BProlog, Mistral, G12/CBC	3698
11	MinisatID, CPX, Fzn2smt, OR-Tools, G12/LazyFD, Gecode, iZplus, BProlog, G12/FD, Mistral, G12/CBC	3698

Table 4.6: Portfolios composition. Solvers are sorted by number of solved instances.

Looking at the overall performances of the single solvers depicted in Fig. 4.5, it can be noted that the best portfolio Π_m is not always the one containing the best m solvers in terms of PSI. Indeed, as already mentioned, an important factor for the success of a portfolio is also the marginal contribution of every constituent solver. Figure 4.6 depicts a measure of the marginal contribution of every solver, showing how many times a solver can solve an instance that no other can solve.

Among all the constituent solvers, we properly elected a backup solver for each portfolio. The choice in this case fell on MinisatID [50][14]

[14] We elected the backup solver by using different voting criteria, namely *Borda*, *Approval* and *Plurality*. The winner, MinisatID, is also the solver that solves the greatest number of instances within the time limit T and has the biggest marginal contribution. In the following, we will refer to it also as the Single Best Solver (SBS).

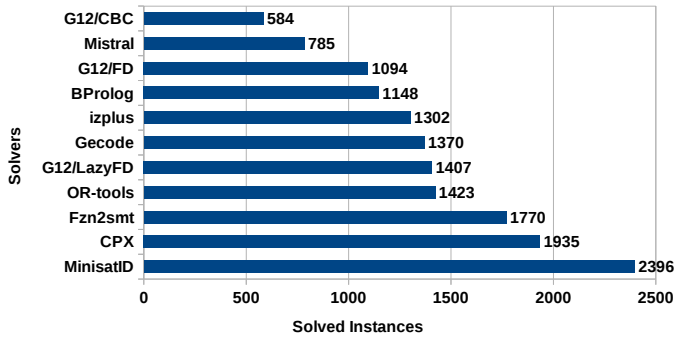


Figure 4.5: Total number of solved instances for each solver of the portfolio.

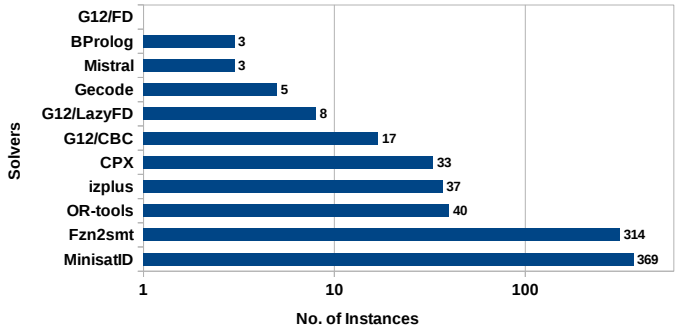


Figure 4.6: Number of times a solver can solve an instance not solvable by any other solver. Please note the logarithmic scale of the x axis.

4.2.2.3 Features

In Section 4.1.1.3 we exploited the features computed by Mistral solver, plus some additional features. The point is that such features are extracted from a XCSP specification. Since in this evaluation we switched to MiniZinc, the Mistral extractor was no longer suitable.

In order to extract a proper set of features starting from a MiniZinc model, we developed `mzn2feat`. This tool allows to compute a set of 155 different features starting from a generic MiniZinc model. Most of these features (144) are static, while 11 of them are dynamic (obtained by running Gecode solver for two seconds).

Although some of the features extracted by `mzn2feat` are quite generic (e.g., the number of variables or constraints), others are bound to FlatZinc language [22]

(e.g., search annotations) or to Gecode (the global constraints features and the dynamic features). An overview of the features extracted by `mzn2feat` is however summarized in Table 4.7 where we denote with NV the number of variables, with NC the number of constraints of a given problem, with min, max, avg, CV, and H respectively the minimum, maximum, average, variation coefficient and the entropy of a set of values. We tried to collect a set of features as large and general as possible, obtaining a number of features that is more than triple of the Mistral feature set introduced in Section 4.1.1.3. Obviously, not all of these features are equally significant. We reiterate that a depth analysis and evaluation of the features is not crucial part of this thesis. For more details about `mzn2feat` technicalities, please see [145, 22, 74].

Typically, not all of the features that `mzn2feat` computes are equally significant. Their importance in fact depends on the dataset and on the prediction algorithm used for the solver selection. For example, if none of the problems contains float variables or objective functions, then the corresponding features have no discriminant power and can be safely discarded. We thus exploited `mzn2feat` for extracting a feature vector $\langle f_1, f_2, \ldots, f_{155} \rangle \in \mathbb{R}^{155}$ from every problem of the initial dataset. Following what is usually done by most of the current approaches, we removed all the constant features from our features set and we scaled their values in the reduced range $[-1, 1]$. In this way, we associated to each problem a feature vector $\langle f'_1, f'_2, \ldots, f'_{114} \rangle \in [-1, 1]^{114}$ consisting of 114 normalized features.

After the features extraction, we also filtered the initial dataset of 8600 instances described in Section 4.2.2.1 by removing the "easiest" ones (i.e., those solved by Gecode during the feature extraction when computing the dynamic features) and the "hardest" ones (i.e., those for which the features extraction required more than half of the timeout). We discarded these instances essentially for two reasons. First, if an instance is already solved during the features extraction then no prediction is needed. Second, if the feature extraction time exceeds half of the timeout it is reasonable to assume that the recompilation of the MiniZinc model into the FlatZinc format would end up in wasting the whole available time to solve the instance. The

Variables (27): the number of variables NV; the number cv of constants; the number av of aliases; the ratios $\frac{av+cv}{NV}$ and $\frac{NV}{NC}$; the number of *defined* variables; the number of *introduced* variables; the logarithm of the product of the: variables domain and variables degree; sum, min, max, avg, CV, and H of the: variables domain size, variables degree, domain size to degree ratio.

Domains (18): the number of:
boolean variables bv and the ratio $\frac{bv}{NV}$;
float variables fv and the ratio $\frac{fv}{NV}$;
integer variables iv and the ratio $\frac{iv}{NV}$;
set variables sv and the ratio $\frac{sv}{NV}$;
array constraints ac and the ratio $\frac{ac}{NC}$; boolean constraints bc and the ratio $\frac{bc}{NC}$; int constraints ic and the ratio $\frac{ic}{NC}$;
float constraints fc and the ratio $\frac{fc}{NC}$;
set constraints sc and the ratio $\frac{sc}{NC}$.

Constraints (27): the total number of constraints NC, the ratio $\frac{NC}{NV}$, the number of constraints using specific FlatZinc annotations (6); the logarithm of the product of the: constraints domain and constraints degree; sum, min, max, avg, CV, and H of the: constraints domain, constraints degree, domain to degree ratio.

Global Constraints (29): the total number gc of global constraints, the ratio $\frac{gc}{NC}$ and the number of global constraints for each one of the 27 equivalence classes in which we have grouped the 47 global constraints that Gecode natively supports.

Graphs (20): once built the Constraint Graph CG and the Variable Graph VG we compute min, max, avg, CV, and H of the: CG nodes degree, CG nodes clustering coefficient, VG nodes degree, VG nodes diameter.

Solving (11): the number of *labelled* variables (i.e. the variables to be assigned); the solve goal; the number of search annotations (3); the number of variable choice heuristics (3); the number of value choice heuristics (3).

Objective (12): the domain dom, the degree deg, the ratios $\frac{dom}{deg}$ and $\frac{deg}{NC}$ of the variable v that has to be optimized; the degree de of v in the variable graph, its diameter di, $\frac{de}{di}$, and $\frac{di}{de}$. Moreover, named μ_{dom} and σ_{dom} the mean and the standard deviation of the variables domain size and μ_{deg} and σ_{deg} the mean and the standard deviation of the variables degree, we compute $\frac{dom}{\mu_{dom}}$, $\frac{deg}{\mu_{deg}}$, $\frac{dom-\mu_{dom}}{\sigma_{dom}}$, and $\frac{deg-\mu_{deg}}{\sigma_{deg}}$.

Dynamic (11): the number of solutions found; the number p of propagations performed; the ratio $\frac{p}{NC}$; the number e of nodes expanded in the search tree; the number f of failed nodes in the search tree; the ratio $\frac{f}{e}$; the maximum depth of the search stack; the peak memory allocated; the CPU time needed for converting from MiniZinc to FlatZinc; the CPU time required for static features computation; the total CPU time needed for extracting all the features.

Table 4.7: Features extracted by `mzn2feat`.

final dataset $\mathbb{D}$ was thus constituted by 4642 instances (3538 from ICSC, 6 from MiniZinc Challenge 2012, and 1098 from MiniZinc 1.6 benchmarks).

4.2.3 Results

In this section we present the experimental results of this second evaluation. In addition to the 3S, CPHydra, ISAC[15], and SATzilla based approaches introduced in Section 4.1.1.7 we added to the comparison also the SUNNY approach showed in Section 4.2.1.

Regarding the OTS approaches, we used the WEKA tool for experimenting different techniques like oversampling, parameters tuning, meta-classifiers, and feature selection. Unfortunately, this has not led to particularly meaningful improvements: all the performance gains were always below 1%. We also tried to apply the filtering techniques used in [124] but this has proved to be too time consuming (i.e., days of computation). In this section we report only the two best OTS approaches, namely Random Forest (RF) and SMO. The RF algorithm uses 250 decision trees, while for SMO we got the best results with a RBF kernel and the parameters set to $C = 2^9$ and $\gamma = 2^{-8}$.

In order to avoid discrepancies and biases, for all the other approaches using a k-NN algorithm (viz., 3S, CPHydra, ISAC, and SUNNY) we fixed $k = 10$ and used the Euclidean distance as distance metric. As done in Section 4.1, for CPHydra approach we did not take into account the time needed for computing the solvers schedule. We evaluated the performances of every approach in terms of AST and PSI by using a time cap of $T = 1800$ seconds and a 5-repeated 5-fold cross validation. In addition to the portfolio approaches, we included also the SBS and VBS performances.

Before presenting the results, let us dwell for a while on the ideal performances of the perfect portfolio approach. The plot in Fig. 4.7 represents the solving time

[15] ISAC is the only approach for which we employed a new version w.r.t. the one presented in Section 4.1. Precisely, we used the version 2.0 downloadable at `http://4c.ucc.ie/~ymalitsky/Code/ISAC-Portfolio_v2.zip`.

of the VBS for each instance of $\mathbb{D}$ (for readability, the instances on the `x` axis are sorted by increasing solving time).

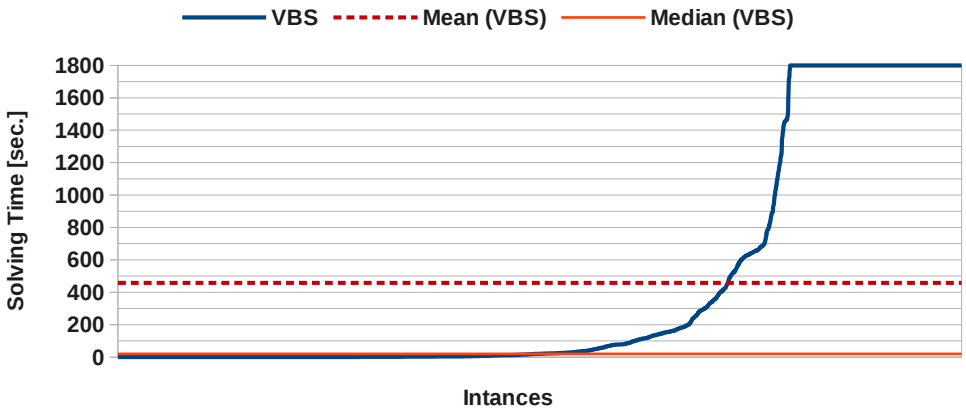


Figure 4.7: Runtimes distribution of VBS on all the instances of the dataset $\mathbb{D}$.

The graph clearly shows the heavy-tailed nature of combinatorial search: most of the problems are either solved in a matter of seconds, or are not solved at all within the timeout. This is also substantiated by the mean values: the average solving time is 458.03 seconds with a high standard deviation of 716.7 seconds. The median is instead 20.45 seconds. This means that for at least half of the instances of $\mathbb{D}$ there exists a solver able to solve them in less than half a minute.

4.2.3.1 Percentage of Solved Instances

Figure 4.8 shows the PSI reached by the various approaches. It can be noted that all the approaches have good performance, and somehow the plot reflects what was already been observed in the previous experiments. In particular, as also shown in Section 4.1, 3S and SATzilla turn out to be better than the OTS approaches. However, the latter seem to take advantage of the new set of solvers, problems, and features being able to outperform ISAC and CPHydra. Moreover, in these experiments there is no the performance deterioration observed in Section 4.1 when increasing the portfolio size. The addition of a new solver to the portfolio is almost always beneficial, or at least not so harmful. The peak performances are reached

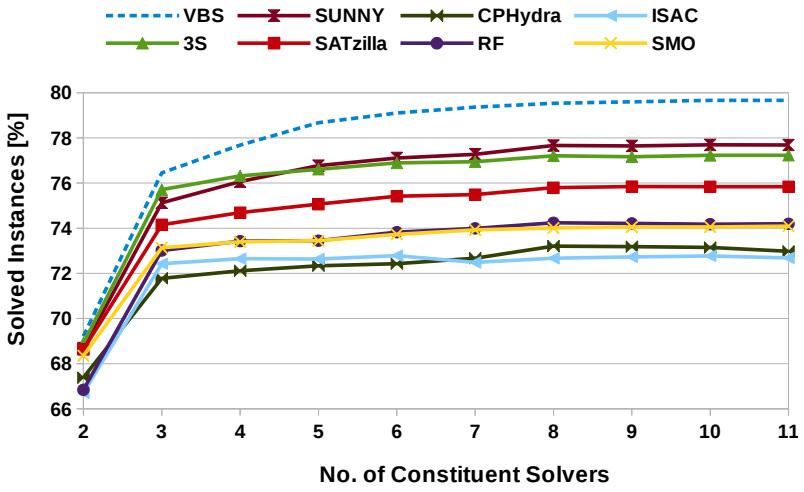


Figure 4.8: PSI results considering all the approaches and the VBS.

by 3S and SUNNY while in this case SATzilla is slightly worse. Figure 4.9 depicts the performance of 3S and SUNNY together with the VBS and the SBS. What is immediately visible is the considerable difference in performance w.r.t. the single best solver MinisatID. Indeed, while the SBS solves just 51.62% of the instances, the peak performances of SUNNY and 3S (reached with 10 and 11 solvers) are respectively 77.23% and 77.69%. In particular, SUNNY is able to close up to 92.95% of the gap between the SBS and the VBS. The peak performance of the other approaches are instead: SATzilla 75.85% (with 9 solvers), RF 74.24% (8 solvers), SMO 74.09% (11 solvers), CPHydra 73.21% (8 solvers), ISAC 72.79% (6 solvers).

According to the *Student's t-test* [97], all the peak performances are statistically significant.

4.2.3.2 Average Solving Time

Figure 4.11 shows the Average Solving Time for each approach. The plot substantially reflects the results reported in Section 4.1. Also in this case, for all the approaches except CPHydra, the AST is pretty anti-correlated with the PSI. It is worth noticing that, unlike what happened in the previous experiments, 3S turns out to be slower on average. This is due to the use of different problem instances and

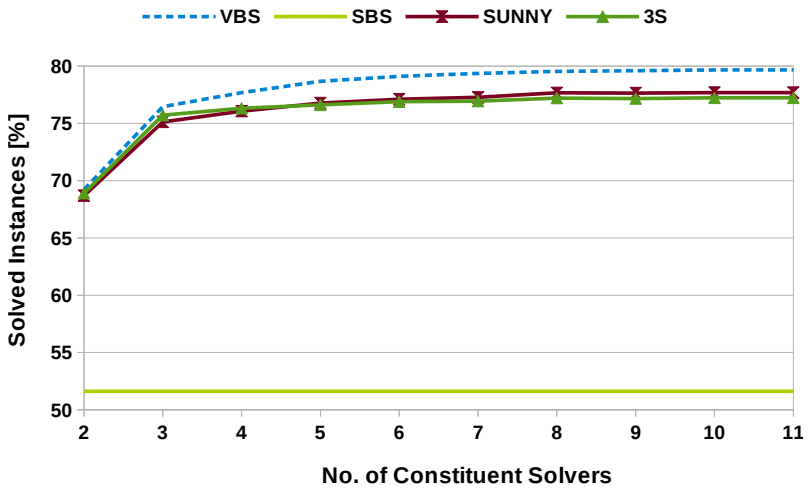


Figure 4.9: PSI results considering SBS, VBS, and the best two approaches.

Figure 4.10: PSI performances.

solvers, but also to the lack in 3S of an heuristic for deciding the execution order of the scheduled solvers. However, on average 3S results faster than CPHydra because in 3S the solvers schedule is limited to the first 180 seconds and it solves a larger number of instances (recall that if a solver is not able to solve a given instance, its solving time is set to $T = 1800$ seconds). The heuristic used by SUNNY (the scheduled solvers are sorted by increasing solving time in the neighbourhood) allows it to minimize the AST and to be the overall best approach. SATzilla performances are not so far from SUNNY, confirming that it minimizes the AST more than 3S even when it solves less instances.

Also in this case, there is a remarkable difference between the SBS (950.91 seconds) and the best approaches (see Figure 4.12). The best AST performance, reached by SUNNY using 10 solvers (568.84 seconds), is able to close the 77.52% of the gap between the SBS and the VBS. Regarding ISAC, RF, and SMO, we can notice the same anti-correlation with the PSI evidenced in Section 4.1. Considering all the reported approaches, with the exception of CPHydra, the strong anti-correlation is confirmed by the Pearson product-moment coefficient (about -0.79). There is instead a strong correlation (about 0.96) between the PSI and AST of CPHydra.

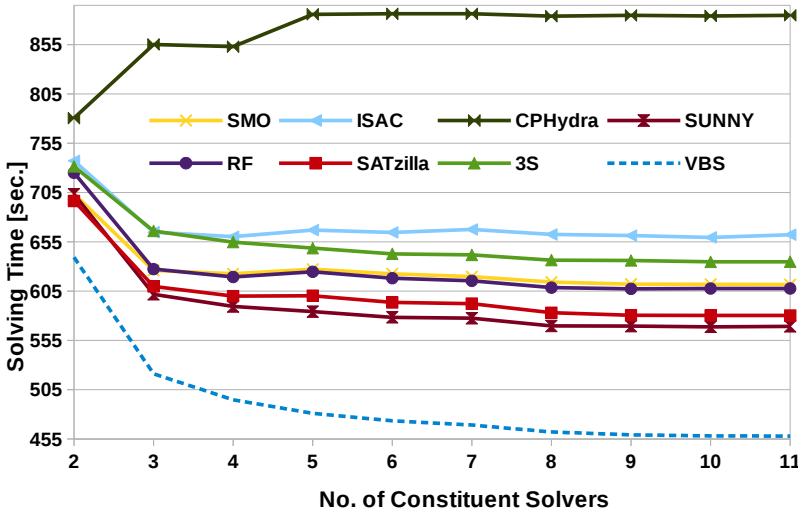


Figure 4.11: AST results considering all the approaches and the VBS.

Nonetheless, in this case the worst performance of CPHydra (884.81 seconds) however outperforms the SBS. Even it this case, all the peak performances in terms of AST are statistically significant according with the Student's t-test.

4.2.3.3 `sunny-csp`

The experimental results indicate that among the tested approaches the best performance in terms of AST and PSI is reached by SUNNY. Moreover, this approach also appears to be quite robust when varying the neighbourhood size k. Indeed, as depicted in Fig. 4.14, varying the value of k in $[5, 20]$ does not entail a huge impact in performance (i.e., less than 1% of solved instances). The peak performance ($PSI = 77.81\%$) is reached with $k = 16$, while for $k > 20$ we observed a gradual performance degradation. For instance, by using $k = 25, 50, 100, 250, 500$ we get a maximum PSI of $77.59, 77.55, 77.3, 77.16, 76.56$, and 72.63 respectively (see Figure 4.15). Of course, the robustness of SUNNY also depends on other factors: for instance, by using different sets of solvers or instances these assessments may be no longer be true.

However, we remark that the results so far described are based on simulations.

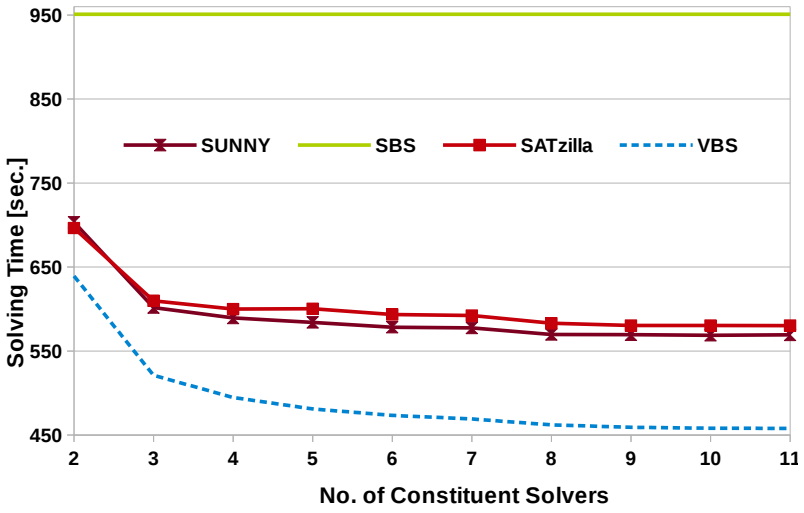


Figure 4.12: AST results considering SBS, VBS, and the best two approaches.

Figure 4.13: AST performances.

Apart from the already mentioned issues in using the original approaches, we decided to simulate the execution of each approach because running every portfolio solver on each fold/repetition would take a tremendous amount of time. For example, every single approach would have to be validated on $4547 \cdot 5 = 22735$ problems in the first evaluation and $4642 \cdot 5 = 23210$ problems in this evaluation. Clearly, it is legitimate to ask if these simulations are faithful. Discrepancies may arise from factors not considered in the simulations like, for instance, the presence of memory leaks, solvers erratic behaviours, or solver faults. Moreover, since some of the features are dynamic, executing again the feature extraction process may lead to slightly different features that may however cause a deviance on the expected performance.

Driven by these motivations, we therefore decided to develop and test `sunny-csp`: a CSP portfolio solver built on top of SUNNY algorithm. More precisely, `sunny-csp` implements SUNNY by exploiting the features extractor `mzn2feat` described in Section 4.2.2.3, a neighbourhood size $k = 16$ (i.e., the value that allows to reach the peak performance according to Figures 4.15 and 4.14), a portfolio of the eleven solvers listed in Section 4.1.1.2, and MinisatID as a backup solver. If a scheduled

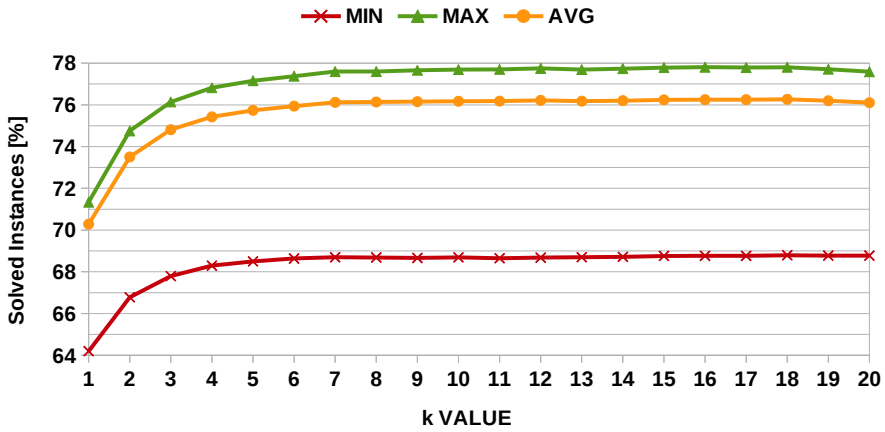


Figure 4.14: Minimum, maximum, and average PSI of SUNNY on all the given portfolios, by varying the k parameter in $\{1, 2, \ldots, 20\}$.

solver prematurely terminates its execution, `sunny-csp` assigns the remaining time to the next scheduled one (if any).

In order to measure and compare the actual performance of `sunny-csp` w.r.t. its simulated performance we used the very same training/test sets, timeout, and machines involved in the simulations. Tables 4.8 and 4.9 report a comparison between the simulated (i.e., ideal) performance of SUNNY and the actual performance of `sunny-csp` for each repetition and fold in terms of PSI and AST respectively. As one can see, the real performance are very close to the expected ones. In particular, `sunny-csp` solves on average only 0.07% instances less than predicted (see Table 4.8). There are cases in which it is also better than expected (see repetition 4 of Table 4.8).

Looking at the AST statistics (Table 4.9), we noted a substantial equivalence between the two approaches (even if on average `sunny-csp` is slightly better, a mean difference of 1.78 seconds appears quite insignificant). We can therefore say that the `sunny-csp` performance are basically equivalent to the expected peak performance of SUNNY, thus witnessing the reliability of our simulations. The source code of `sunny-csp` is available at `http://www.cs.unibo.it/~amadini/iclp_2014.zip`.

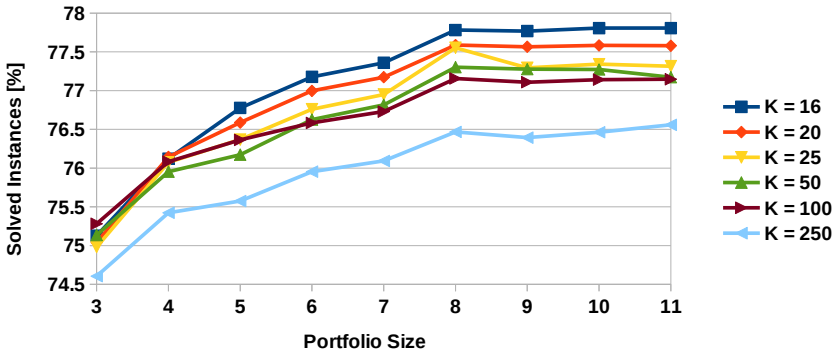


Figure 4.15: PSI of SUNNY by varying $k \in \{16, 20, 25, 50, 100, 250\}$.

4.2.3.4 Training Cost

Although we evaluated the performances of different CSP portfolio solvers in terms of PSI and AST, as done in Section 4.1 we would like to discuss shortly also the time needed to build a prediction model. This time is not that significant and can be omitted when measuring the actual performances of a portfolio, since the prediction model is built offline (maybe by using powerful clusters to parallelise the workload). However this could be no longer true when considering more dynamic scenarios, like case-based reasoning systems that dynamically exploit new incoming knowledge to increase their efficiency and accuracy. Obviously, all the training-based approaches we considered might be adapted to dynamic scenarios by periodically updating the models. However, if updating a model requires a long time (e.g., hours) the system may be not very responsive in a dynamic context. Furthermore, some of the main complaints about portfolios concern their poor flexibility and usability: if for winning a competition a portfolio solver needs a year of training, it is clear that outside a competition setting its use will result very limited.

In Figure 4.16 we report the times needed by different approaches to build the prediction model of a single fold (928 instances) of the dataset.[16] We do not consider the lazy approaches CPHydra and SUNNY since they do not need to build and train

[16]We remark that these times are just an estimation since for some approaches like 3S and SATzilla we were forced to develop a customized version. Moreover, note that such measurements could vary (especially for approaches like 3S) according to the particular fold considered.

		IDEAL		ACTUAL		ACTUAL − IDEAL	
Rep.	**Fold**	**PSI**	***AVG***	**PSI**	***AVG***	**PSI**	***AVG***
1	1	75.78		75.24		−0.54	
1	2	78.69		78.15		−0.54	
1	3	78.77		78.77		0.00	
1	4	78.13		78.23		0.11	
1	5	78.23	*77.92*	78.13	*77.70*	−0.11	*−0.22*
2	1	78.90		79.22		0.32	
2	2	76.64		76.53		−0.11	
2	3	78.77		78.77		0.00	
2	4	78.56		77.80		−0.75	
2	5	76.08	*77.79*	76.08	*77.68*	0.00	*−0.11*
3	1	78.79		78.69		−0.11	
3	2	76.32		76.32		0.00	
3	3	78.56		78.13		−0.43	
3	4	78.13		78.66		0.54	
3	5	77.37	*77.83*	77.16	*77.79*	−0.22	*−0.04*
4	1	78.04		78.04		0.00	
4	2	75.35		75.78		0.43	
4	3	78.66		78.56		−0.11	
4	4	78.23		78.23		0.00	
4	5	78.02	*77.66*	77.91	*77.70*	−0.11	*0.04*
5	1	77.93		78.04		0.11	
5	2	78.36		78.47		0.11	
5	3	77.05		76.94		−0.11	
5	4	78.56		78.45		−0.11	
5	5	77.26	*77.83*	77.05	*77.79*	−0.22	*−0.04*
		Total Average	***77.81***	***Total Average***	***77.73***	***Total Average***	***−0.07***

Table 4.8: Percentage of Solved Instances comparison.

an explicit model.

Among the tested methods the one that employs the longest time to build a model is SATzilla. Even though this task can be easily parallelisable, the computation for a portfolio with 11 solvers using only one machine required more than 10 minutes (about 621.55 seconds). From Figure 4.16 it can be noted that the building time for SATzilla grows quadratically w.r.t. the portfolio size, since this approach needs to compute for every pair of solvers a weighted random forest of decision trees.

The time needed by ISAC is instead a kind of constant but not negligible (about 400 seconds). In contrast, the time of SMO is anti-monotonic: from 102.58 to 68.29 seconds. RF instead lightly grows from 30.09 to 36.43 seconds.

As far as the 3S approach is concerned, in Figure 4.16 we present the times needed to compute the static schedule for one fold. As already stated, this is an NP-hard problem and its solution times may heavily depend on the specific training data. However, for the considered fold the scheduling time turned out to be pretty

		IDEAL		ACTUAL		ACTUAL − IDEAL	
Rep.	**Fold**	**AST**	***AVG***	**AST**	***AVG***	**AST**	***AVG***
1	1	610.03		618.39		8.37	
1	2	561.75		567.91		6.16	
1	3	545.77		549.88		4.11	
1	4	550.57		545.54		−5.04	
1	5	571.98	*568.02*	572.29	*570.80*	0.31	*2.78*
2	1	564.72		559.29		−5.43	
2	2	577.05		581.86		4.81	
2	3	540.97		537.76		−3.21	
2	4	557.87		565.39		7.52	
2	5	608.41	*569.80*	602.52	*569.36*	−5.89	*-0.44*
3	1	561.18		557.79		−3.39	
3	2	609.80		607.50		−2.30	
3	3	564.97		568.77		3.80	
3	4	556.06		542.47		−13.60	
3	5	560.65	*570.53*	555.87	*566.48*	−4.78	*-4.05*
4	1	566.71		571.09		4.38	
4	2	599.00		589.31		−9.68	
4	3	551.58		549.03		−2.55	
4	4	581.12		578.40		−2.71	
4	5	552.34	*570.15*	549.28	*567.42*	−3.06	*-2.73*
5	1	571.06		566.74		−4.32	
5	2	558.51		557.11		−1.41	
5	3	580.12		579.47		−0.66	
5	4	581.40		570.17		−11.23	
5	5	568.84	*571.99*	564.09	*567.51*	−4.75	*-4.47*
		Total Average	***570.10***	***Total Average***	***568.32***	***Total Average***	***−1.78***

Table 4.9: Average Solving Time comparison.

low, i.e., between 1.38 and 15.26 seconds.

4.2.3.5 Summary

In this second evaluation we presented a more in-depth analysis compared to that introduced in Section 4.1.

We improved on the previous work by adding several new solvers and by increasing the number of features, problem instances, and algorithms considered. In particular we introduced in the evaluation also the SUNNY algorithm which, despite its simplicity, appears to be very promising. To get a more realistic and significant comparison we also implemented `sunny-csp`, a CSP portfolio solver built on top of SUNNY.

The new results confirmed the effectiveness of CSP portfolios, both in terms of solved instances and solving time. In particular, the simulations show that state-of-the-art approaches like 3S and SATzilla, as well as our new algorithm SUNNY,

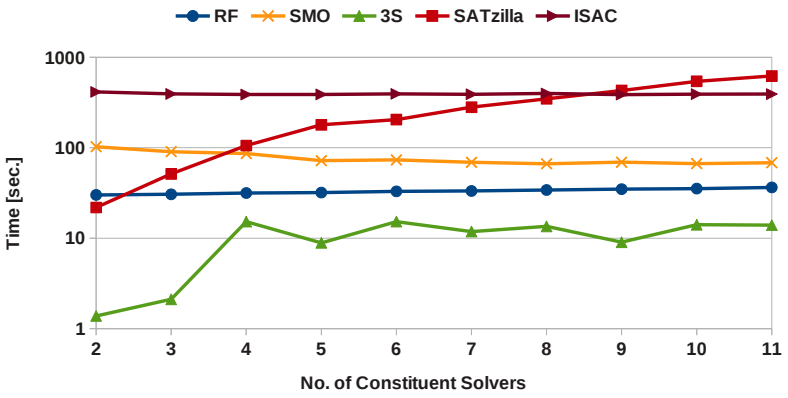


Figure 4.16: Off-line computation times. Note the logarithmic scale of the y-axis.

yield very good results.

We can say that this enhanced evaluation has lead to two main contributions. On the one hand, it has confirmed most of what already observed in the first empirical evaluation. This is not an improvement on the state of the art, but it is important for having a solid baseline.

On the other hand, the new evaluation has also provided new contributions. Firstly, we showed that also lazy approaches like SUNNY can be competitive when compared to the best non-lazy approaches. Furthermore, we developed new tools like `xcsp2mzn`, `mzn2feat`, and `sunny-csp` to ease the task of building and testing of new generation CSP portfolio solvers relying on (also) MiniZinc format. Third, and maybe more important, the new road that we have undertaken allowed us to extend the work to the optimization problems. This promising direction will be discussed more in detail in the next chapters.

Chapter 5

Portfolios of COP Solvers

"Nil Satis Nisi Optimum"[1]

In the previous chapter we empirically verified the benefits of using portfolio solvers for CSPs. Unfortunately, despite the effectiveness of these approaches for satisfaction problems, only a few studies have tried to apply portfolio techniques to COPs. In the 2008 survey on algorithm selection procedures [171] the author observes that: *"there have been surprisingly few attempts to generalize the relevant meta-learning ideas developed by the machine learning community, or even to follow some of the directions of Leyton-Brown et al. in the constraint programming community"*. To the best of our knowledge, we think that the situation has not improved significantly. In the literature just a few works deal with portfolios of COP solvers, referring in particular to a specific problem (e.g., the *Traveling Salesman Problem* [108]) or to a specific solver (e.g., by tuning its parameters [189]).

Clearly, by definition, a COP is more general than a CSP. Some issues which are obvious for CSPs are less clear for COPs. For example, defining a suitable metric which allows to compare different solvers is not immediate. These difficulties explain in part the lack of exhaustive studies on portfolios consisting of different COP solvers. Nevertheless, the COP field is clearly of particular interest. A major contribution of this thesis is therefore the generalization of portfolio approaches from CSPs to COPs. This chapter reports in particular two main investigations.

[1] "Nothing but the best is good enough".

In Section 5.1 we show a first step towards the definition of COP portfolios. Basically, we first formalized a suitable model for adapting the "classical" satisfaction-based portfolios to address COPs, providing also some metrics to measure the solver performance. Then, analogously to what done in Chapter 4 for CSPs, we simulated and evaluated the performances of different COP portfolio solvers.

Similarly to Chapter 4, in Section 5.1 results are based on simulations. The point is that a crucial difference from CSPs arises because a COP solver may yield *sub-optimal* solutions before finding the best one (and possibly proving its optimality). This means that a COP solver can transmit useful bounds information to another one when more than one solver is selected for solving a given COP. In Section 5.2 we therefore take a step forward by addressing the problem of boosting optimization by exploiting the sequential cooperation of different COP solvers.

5.1 An Empirical Evaluation of COP Portfolio Approaches

In this section we tackle the problem of defining and evaluating COP portfolios. In Section 5.1.1 we first formalize a suitable model for generalizing portfolio approaches to COP setting. Then, by making use of an exhaustive benchmark of 2670 COP instances, we report the performances of several portfolio approaches having different size (from two up to twelve constituent solvers). We decided to adapt to the optimization field the best practices according to the results presented in Chapter 4, namely: SATzilla, 3S, SUNNY. In addition, we included in the evaluation some OTS approaches. The methodology and the portfolio algorithms we use to conduct the tests are shown in Section 5.1.2.

The results detailed in Section 5.1.3 indicate that, also in the case of COPs, the reported portfolio approaches almost always significantly outperform the Single Best Solver available. In particular, we observe that the generalization of SUNNY to COPs appears to be particularly effective: this algorithm has indeed reached the best performances in our experiments.

This section is an extension of an our previous work described in [9].

5.1.1 Evaluating COP solvers

When satisfaction problems are considered, the definition and the evaluation of a portfolio solver is pretty straightforward. Indeed, the outcome of a solver run for a given time on a certain instance can be either *'solved'* (i.e., a solution is found or the unsatisfiability is proven) or *'not solved'* (i.e., the solver does not say anything about the problem). Building and evaluating a CSP solver is then conceptually easy: the goal is to maximize the number of solved instances, solving them as fast as possible. Unfortunately, in the COP world the dichotomy solved/not solved is no longer suitable. A COP solver in fact can provide sub-optimal solutions or even give the optimal one without being able to prove its optimality. Moreover, in order to speed up the search, COP solvers could be executed in a non-independent way. Indeed, the knowledge of a sub-optimal solution can be used by a solver to further prune its search space and speed up the search process.

Given the `proven` metric typically used in the CSP field for measuring the number of solved instances (see Def. 4.1) it is possible to get a trivial generalization by considering the number of *optima proven*, that is:

Definition 5.1 (`proven`) *Let us fix a dataset $\mathbb{D}$ of COPs, a set $\mathbb{S}$ of COP solvers, and a solving timeout T. Given a problem $p \in \mathbb{D}$ and a solver $s \in \mathbb{S}$, we define the binary function* $\texttt{proven} : \mathbb{S} \times \mathbb{D} \to \{0, 1\}$ *as:*

$$\texttt{proven}(s, p) := \begin{cases} 1 & \text{if solver } s \text{ proves the optimality of } p \text{ in less than } T \text{ seconds;} \\ 0 & \text{otherwise} \end{cases}$$

Note that the concise expression "s proves the optimality of p" actually means "s finds an optimal solution for p and proves its optimality (including proving unsatisfiability or unboundedness, in case)". However, the significance of such a metric is rather limited since it does not take into account the time taken by a solver to prove the optimality and it excessively penalizes a solver that finds the optimal value

(even instantaneously) without being able to prove its optimality. One can think to adapt the `time` metric defined in Section 4.1.1.4 by measuring the *optimization time* instead of the solving time, i.e.:

Definition 5.2 (`time`) *Let us fix a dataset $\mathbb{D}$ of COPs, a set $\mathbb{S}$ of COP solvers, and a solving timeout T. Given a problem $p \in \mathbb{D}$ and a solver $s \in \mathbb{S}$, we define the function $\texttt{time} : \mathbb{S} \times \mathbb{D} \rightarrow [0, T]$ as:*

$$\texttt{time}(s, p) := \begin{cases} t & \text{if solver } s \text{ proves the optimality of } p \text{ in } t < T \text{ seconds;} \\ T & \text{otherwise.} \end{cases}$$

Unfortunately, even this metric is rather poor discriminating: for most of the non-trivial optimization problems no solver may be able to prove the optimality within a reasonable timeout. The main issue is that, although the ideal goal of a COP solver is to prove the optimality as soon as possible, for many real life applications it is far better to get a good solution in a relatively short time rather than consume too much time to find the optimal value (or proving its optimality). So, the question is: how to properly evaluate a COP solver?

An interesting method for ranking solvers is used in the MiniZinc Challenge [139]. The evaluation metric is based on a *Borda* count voting system [42], where each problem is treated like a voter who ranks the solvers. Each solver gets a score proportional to the number of solvers it beats. A solver s scores points on problem p by comparing its performance with each other solver s' on problem p. If s gives a better answer than s' then it scores 1 point, if it gives a worse solution it scores 0 points. If s and s' give indistinguishable answers then the scoring is based on the optimization time with a timeout of 900 seconds. In particular, s scores 0 if it fails to find any solution or fails to prove unsatisfiability, 0.5 if both s and s' complete the search in 0 seconds. Otherwise the score assigned to the solver s is $\texttt{time}(p, s')/(\texttt{time}(p, s) + \texttt{time}(p, s'))$. This metric takes into account both the best solution found and the time needed for completing the search process, but it has also some disadvantages. In case of indistinguishable answers, it overestimates small time differences for easy instances, as well as underrate big time differences in case

of medium and hard instances. Indeed, let us suppose that two solvers s and s' solve a problem p in 1 and 2 seconds respectively. The score assigned to s will be 2/3 while s' scores 1/3. However, the same score would be reached by the two solvers if $\texttt{time}(p, s') = 2\,\texttt{time}(p, s)$. Hence, if for example $\texttt{time}(p, s) = 400$ and $\texttt{time}(p, s) = 800$, the difference between the scores of s and s' would be the same even if the absolute time difference is one second in the first case and 400 seconds in the second. Moreover, the Borda comparisons of MiniZinc Challenge metric do not take into account the difference in quality between distinguishable answers: the solver that gives the worse solution always scores 0 points, regardless of whether such solution is very close or very far from the best one.

Given these problems, in order to study the effectiveness of COP solvers (and consequently the performance of COP portfolio solvers) we need different and more sophisticated evaluation metrics. In the rest of this section we therefore provide an alternative scoring system.

5.1.1.1 The `score` function

In order to take into account the quality of the solutions without relying on cross comparisons between solvers, we propose the `score` evaluation function. The idea of `score` is to evaluate the solution quality at the stroke of the solving timeout T, by giving to each COP solver (portfolio based or not) a reward proportional to the distance between the best solution it finds and the best known solution. An additional reward is assigned if the optimality is proven, while a punishment is given if no solution is found without proving unsatisfiability or unboundedness. Given a COP instance p, we assign to a solver s a score of 1 if it proves optimality for p, 0 if s gives no answer. Otherwise, we give to s a score corresponding to the value of its best solution scaled into the range $[\alpha, \beta]$ where α and β are two arbitrary parameters such that $0 \leq \alpha \leq \beta \leq 1$. Intuitively, the parameters α and β map respectively the value of the worst and the best known solution of the known COP solvers at the timeout T.

In order to formally define this scoring function and to evaluate the quality

of a solver, we need some preliminary definitions. Let $\mathbb{S}$ be the set of the available solvers (possibly including portfolio solvers), $\mathbb{D}$ a dataset of COP problems and T the solving timeout. We use the function $\texttt{sol}(s,p,t) : \mathbb{S} \times \mathbb{D} \times [0,T] \to \{\texttt{unk}, \texttt{sat}, \texttt{opt}\}$ to map the outcome of a solver s for a problem p at time t. In particular, $\texttt{sol}(s,p,t)$ is either `unk`, if s gives no answer about p; `sat`, if s finds a solution for p without proving the optimality; `opt` if it proves the optimality. Similarly, we use the function $\texttt{val} : \mathbb{S} \times \mathbb{D} \times [0,T] \to \mathbb{R} \cup \{+\infty\}$ to define the values of the objective function. In particular, $\texttt{val}(s,p,t)$ is the best objective value found by solver s for instance p at time t (if no solution is found at that time, $\texttt{val}(s,p,t) = +\infty$). We assume the solvers behave (anti-)monotonically, i.e., as time goes the solution quality gradually improves and never degrades:[2]

$$\forall\ s \in \mathbb{S} : \forall\ p \in \mathbb{D} : \forall\ t, t' \in [0,T] : \quad t \leq t' \implies \texttt{val}(s,p,t') \leq \texttt{val}(s,p,t).$$

We are now ready to define a measure that quantitatively represents how good the solver s is when solving problem p over time t.

Definition 5.3 (`score`) *Let us fix two parameters $\alpha, \beta \in [0,1]$, a dataset $\mathbb{D}$ of COPs, a set $\mathbb{S}$ of COP solvers, and a solving timeout T. We define the scoring value of s (shortly, **score**) on the instance p at a given time t as a parametric function $\texttt{score}_{\alpha,\beta} : \mathbb{S} \times \mathbb{D} \times [0,T] \to [0,1]$ defined as follows:*

$$\texttt{score}_{\alpha,\beta}(s,p,t) := \begin{cases} 0 & \text{if } \texttt{sol}(s,p,t) = \texttt{unk}; \\ 1 & \text{if } \texttt{sol}(s,p,t) = \texttt{opt}; \\ \beta & \text{if } \texttt{sol}(s,p,t) = \texttt{sat} \wedge \min V_p = \max V_p; \\ \max\left\{\beta - (\beta-\alpha) \cdot \dfrac{\texttt{val}(s,p,t) - \min V_p}{\max V_p - \min V_p}, 0\right\} & \text{otherwise.} \end{cases}$$

where $V_p = \{\texttt{val}(s,p,T) \ : \ s \in \mathbb{S}\}$ is the set of the objective function values found by any solver s at the time limit T.[3]

[2] We can omit non-monotonic entry, if any. Recall that, without loss of generality, we consider only minimization problems.

[3] Note that a portfolio solver executing more than one solver for $t < T$ seconds could produce

The score of a solver is therefore a measure in the range $[0, 1]$ that is linearly dependent on the distance between the best solution it finds and the best solutions found by every other available solver. To better clarify the `score` definition, consider the Example 5.1.

Example 5.1 *Let us consider the scenario in Figure 5.1 depicting the performances of three different solvers run on the same minimization problem.*

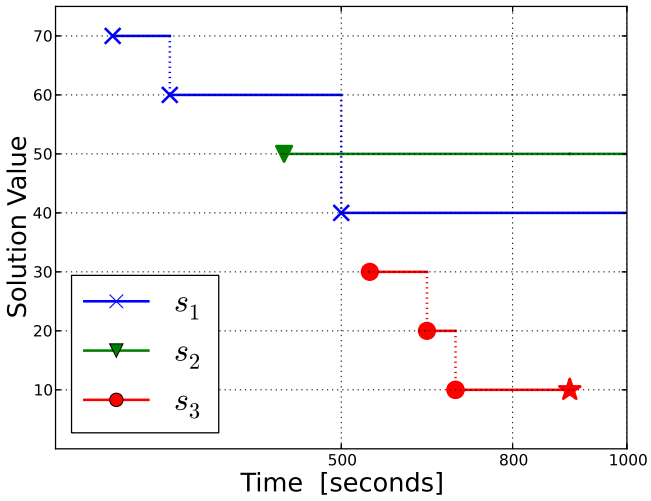


Figure 5.1: Solvers performance example.

By choosing $T = 500$ *as time limit,* $\alpha = 0.25$*, and* $\beta = 0.75$*, the score assigned to* s_1 *is* 0.75 *because it finds the solution with minimum value (40), the score of* s_2 *is* 0.25 *since it finds the solution with maximum value (50), and the score of* s_3 *is 0 because it does not find a solution in* T *seconds. If instead* $T = 800$*, the score assigned to* s_1 *becomes* $0.75 - (40 - 10) \cdot 0.5/(50 - 10) = 0.375$ *while the score of* s_2 *is* 0.25 *and the score of* s_3 *is* 0.75*. If* $T = 1000$*, since* s_3 *proves the optimality of the value 10 at time 900 (see the point marked with a star in Figure 5.1) it receives a corresponding reward reaching then the score 1.*

a solution that is outside the range $[\min V_p, \max V_p]$, potentially generating a score lower than α (and even lower than zero). The latter case however is very uncommon: in our experiments we noticed that the 0 score was assigned only to the solvers that did not find any solution.

Being `score` a parametric function, it obviously also depends on the α and β parameters. Among all the possible choices, we found it reasonable to set $\alpha = 0.25$ and $\beta = 0.75$. In this way, we halved the codomain: one half (i.e., the range $[0.25, 0.75]$) is intended to map the quality of the sub-optimal solutions, while the other half consists of two "gaps" (i.e., the intervals $[0, 0.25]$ and $[0.75, 1]$) that are meant to either penalize the lack of solutions or to reward the proven optimality. In the following, unless otherwise specified, we will simply use the notation `score` (or simply score) to indicate the function $\texttt{score}_{0.25,0.75}$. As one can imagine, other thresholds would have been equally justifiable. Even though a systematic study of the impact of α and β parameters is outside the scope of this work, in the following we will also report the results obtained by using the $\texttt{score}_{0,1}$ function (see Section 5.1.3.1). Furthermore, with a little abuse of notation, we will use the short notation $\texttt{score}(s, p)$ in place of $\texttt{score}(s, p, T)$, where T is the solving timeout.

5.1.2 Methodology

Taking as baseline the methodology and the results of Chapter 4, in this section we present the main ingredients and the procedure that we have used for conducting our experiments and for evaluating the COP portfolios.

5.1.2.1 Solvers, Dataset, and Features

We built the set $\mathbb{S}$ of the single solvers used to define our portfolios by exploiting 12 solvers of the MiniZinc Challenge 2012, namely: BProlog, Fzn2smt, CPX, G12/FD, G12/LazyFD, G12/MIP, Gecode, izplus, JaCoP, MinisatID, Mistral and OR-Tools. We used all of them with their default parameters, their global constraint redefinitions when available, and keeping track of each solution found by every solver within the timeout of the MiniZinc Challenge, i.e., $T = 900$ seconds.

To conduct our experiments on a dataset of instances as realistic and large as possible, we have considered all the COPs of the MiniZinc 1.6 benchmark [146]. We also added all the instances of the MiniZinc Challenge 2012 thus obtaining an initial

dataset of 4977 instances in MiniZinc format. In order to reproduce the portfolio approaches, we have extracted for each instance a set of 155 features by exploiting the features extractor `mzn2feat` introduced in Section 4.2.2.3. We preprocessed these features by scaling their values in the range $[-1, 1]$ and by removing all the constant features. In this way, we ended up with a reduced set of 130 features on which we conducted our experiments. We have also filtered the initial dataset by removing, on one hand, the "easiest" instances (i.e., those for which the optimality was proven during the feature extraction) and, on the other, the "hardest" (i.e., those for which the features extraction has required more than T seconds).[4] The final dataset $\mathbb{D}$ on which we conducted our experiments thus consisted of 2670 instances.

5.1.2.2 Portfolio Approaches

After running every solver of $\mathbb{S}$ on each instance of $\mathbb{D}$ keeping track of all the anytime performances, we built portfolios of different size. While in the case of CSPs the ideal choice is typically to select the portfolio that maximizes the (potential) number of solved instances, in our case such a metric is no longer appropriate since we have to take into account the quality of the solutions. We decided to select for each portfolio size $m = 2, 3, \ldots, 12$ the portfolio $\Pi_m \subseteq \mathbb{S}$ of size m that maximizes the total `score` (possible ties have been broken by minimum `time`). Formally:

$$\Pi_m = \operatorname*{arg\,max}_{\Pi \in \{S \subseteq \mathbb{S} \mid |S|=m\}} \sum_{p \in \mathbb{D}} \max\{\texttt{score}(s, p) \mid s \in \Pi\}$$

We then elected CPX (a solver of the MiniZinc suite [146]) as the *backup solver*.[5]

We tested different portfolio techniques. We considered the aforementioned SATzilla and 3S approaches together with some OTS approaches (namely, IBk, J48, PART, RF, and SMO). Moreover, we have also implemented a generalization of

[4] The time needed for extracting features was strongly dominated by the FlatZinc conversion. However, for the instances of the final dataset this time was in average 10.36 seconds, with a maximum value of 504 seconds and a median value of 3.17 seconds.

[5] CPX was selected because it won all the elections we simulated by using *Borda*, *Approval*, and *Plurality* criteria. CPX will be also referred as the *Single Best Solver* (SBS) of the portfolio.

SUNNY in order to deal with COPs. We adapted these approaches trying to modify the original versions as little as possible. Obviously, for 3S, SATzilla, and SUNNY approaches was not possible to use the original solvers since they are tailored for satisfaction problems. In the following we provide an overview of how we implemented such portfolio approaches.

Off-the-shelf As in the case of satisfiability OTS approaches were implemented by simulating the execution of a solver predicted by a ML classifier. Adapting the OTS approaches to the COP context was pretty easy. Following the methodology already described in 4.1.1.6 we built 5 different approaches using well-known ML classifiers, viz.: IBk, J48, PART, Random Forest, and SMO. We exploited their implementation in WEKA with default parameters, and we trained the models by adding for each instance $p \in \mathbb{D}$ a label $l_p \in \mathbb{S}$ corresponding to the best constituent solver w.r.t. the `score` metric.

3S The major issue when adapting 3S for optimization problems is to compute the fixed-time schedule since, differently from SAT problems, in this case the schedule should also take into account the quality of the solutions. We then tested different minimal modifications, trying to be as little invasive as possible and mainly changing the objective metric of the original IP problem used to compute the schedule. The performances of the different versions we tried were very similar. Among those considered, the IP formulation that achieved the best performance is the one that: first, tries to maximize the solved instances; then, tries to maximize the sum of the score of the solved instances; finally, tries to minimize the solving time. Formally, the objective function of the best approach considered was obtained by replacing that of the IP problem defined in [113] (we use the very same notation) by:

$$\max \left[C_1 \sum_{y} y_i + C_2 \sum_{i,S,t} \texttt{score}(S,i,t) \cdot x_{S,t} + C_3 \sum_{S,t} t \cdot x_{S,t} \right]$$

where $C_1 = -C^2$, $C_2 = C$, $C_3 = -\frac{1}{C}$. We added the constraint $\sum_t x_{S,t} \leq 1$, $\forall S$ to avoid selecting the same solver more than one time.

SATzilla Unlike 3S, reproducing this approach turned out to be more straightforward. The only substantial difference concerned the construction of the runtimes

matrix exploited by SATzilla to construct its selector, which is based on $m(m-1)/2$ pairwise cost-sensitive decision forests.[6] Since our goal is to maximize the score rather than to minimize the runtime, instead of using such a matrix we have defined a matrix of "anti-scores" P in which every element $P_{i,j}$ corresponds to the score of solver j on instance i subtracted to 1, that is, $P_{i,j} = 1 - \texttt{score}(j, i)$. In this way, we maintained the invariant property stating that $P_{k,i} < P_{k,j}$ if and only if the solver s_i performs better than s_j on the instance p_k.

SUNNY Even in the case of SUNNY we faced some design choices to tailor the algorithm for optimization problems. In particular, we decided to select the sub-portfolio that maximizes the score in the neighbourhood $N(p, k)$ and we allocated to each solver a time proportional to its total score in $N(p, k)$. While in the CSP version SUNNY allocates to the backup solver an amount of time proportional to the number of instances not solved in $N(p, k)$, here we have instead assigned to it a slot of time proportional to $k - h$ where h is the maximum score achieved by the sub-portfolio. Example 5.2 provides an illustrative example of how SUNNY works on a given COP.

Example 5.2 *Let us suppose that to have a portfolio* $\Pi = \{s_1, s_2, s_3, s_4\}$ *where the backup solver is* s_3*, the timeout is* $T = 1000$ *seconds, the neighbourhood size is* $k = 3$*, the neighbourhood is* $N(p, k) = \{p_1, p_2, p_3\}$*, and the scores/optimization times are defined as listed in Table 5.1.*

	p_1	p_2	p_3
s_1	$(\mathbf{1}, 150)$	$(0.25, 1000)$	$(\mathbf{0.75}, 1000)$
s_2	$(0, 1000)$	$(\mathbf{1}, 10)$	$(0, 1000)$
s_3	$(1, 100)$	$(0.75, 1000)$	$(0.7, 1000)$
s_4	$(0.75, 1000)$	$(0.75, 1000)$	$(0.25, 1000)$

Table 5.1: (`score`, `time`) of each solver s_i for every COP p_j.

The minimum size sub-portfolio that allows to reach the highest score $h = 1+1+0.75 = 2.75$ *is* $\{s_1, s_2\}$*. On the basis of the sum of the scores reached by* s_1 *and* s_2 *in*

[6]For more details, we defer the interested reader to [190].

$N(p,k)$ *(resp. 2 and 1) the slot size is* $t = T/(2+1+(k-h)) = 307.69$ *seconds. The time assigned to* s_1 *is* $2 * t = 615.38$ *while for* s_2 *is* $1 * t = 307.69$*. The remaining* 76.93 *seconds are finally allocated to the backup solver* s_3*. After sorting the solvers by increasing optimization time, SUNNY executes first* s_2 *for* 615.38 *seconds, then* s_3 *for* 76.93 *seconds, and finally* s_1 *for* 307.69 *seconds.*

The algorithm used by SUNNY on COPs is a straight generalization of the algorithm reported in Listing 4.1 for CSPs. More precisely, in this case the function `getMaxSolved`(S, `similar_insts`, `KB`, `T`) returns the maximum score achieved by the sub-portfolio S on `similar_insts`, instead of the number of instances of `similar_insts` that S is able to solve. Also the function `getSubPortfolio` changes accordingly. Formally, fixed a timeout T, a portfolio $\Pi = \{s_1, s_2, \ldots, s_m\}$, and a set of instances $P = \{p_1, p_2, \ldots, p_k\}$, we define an auxiliary function $\psi : \mathcal{P}(\Pi) \rightarrow \{0, 1, \ldots, k\} \times \{0, 1, \ldots, m\} \times [0, T]$ such that, for each $S \subseteq \Pi$:

$$\psi(S) := (k - \sum_{p \in P} \max\{\texttt{score}(s,p) \mid s \in S\}, |S|, \sum_{s \in S, p \in P} \texttt{time}(s,p))$$

where $|S|$ is the cardinality of S and `score`, `time` are defined as in Definitions 5.3 and 5.2 respectively. The function `getSubPortfolio` returns the subset of Π that minimizes w.r.t. the lexicographic ordering the set of triples $\{\psi(S) \mid S \subseteq \Pi\}$.

5.1.3 Results

In this section we present the experimental results achieved by the different approaches, by using a timeout of $T = 900$ seconds and by varying the portfolio size in $[2, 12]$. As in the previous experiments, we used a 5-fold 5-repeated cross validation over the dataset $\mathbb{D}$. To conduct the experiments we used Intel Dual-Core 2.93GHz computers with 3 MB of CPU cache, 2 GB of RAM, and Ubuntu 12.04 Operating System. For keeping track of the optimization times we considered the CPU time by exploiting Unix `time` command. The performances are measured in terms of the above defined metrics: `score`, `proven`, `time`. In addition, we also considered the MiniZinc Challenge score as described in Section 5.1.1.

For ease of reading, in all the plots we report only the two best approaches among all the off the shelf classifiers we evaluated, namely Random Forest (RF) and SMO. As a baseline for our experiments, in addition to the Single Best Solver (SBS) CPX, we have also introduced the Virtual Best Solver (VBS). The source code developed to conduct and replicate the experiments is publicly available at `http://www.cs.unibo.it/~amadini/amai_2014.zip`.

5.1.3.1 Score

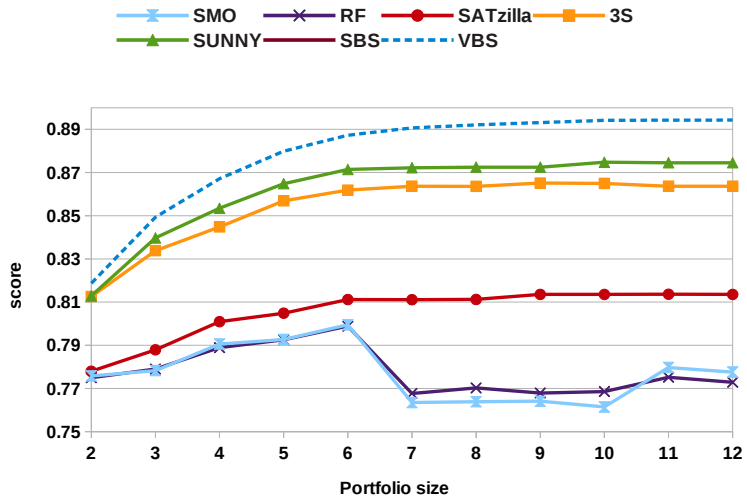
Figure 5.2a shows the average results of `score` by considering all the portfolio approaches and the VBS. For the sake of readability, Figure 5.2b visualizes instead the same results by considering the VBS, the SBS, and the two best approaches only.

As expected, all the considered approaches have good performances and greatly outperform the SBS (even with portfolios of small size). This is encouraging, since the portfolio composition and the portfolio approaches were defined with the aim of maximizing the `score` function.

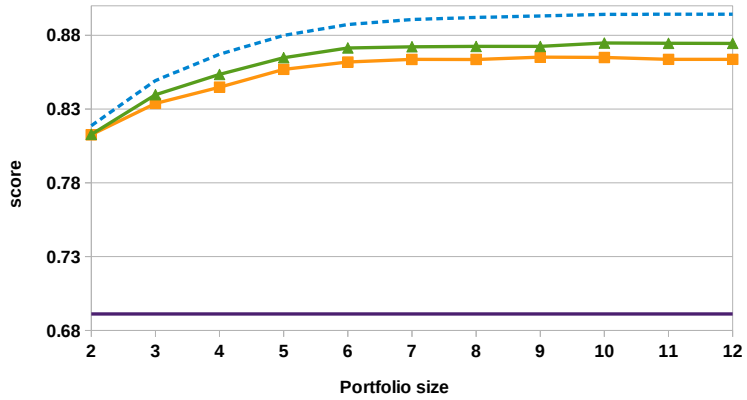
Similarly to what happens to the Percentage of Solved Instances for CSP portfolios (see empirical results in Chapter 4), we notice that the OTS approaches have usually lower performances, even though the gap between the best approaches and them is not so pronounced.

The best portfolio approach is SUNNY, that reaches a peak performance of 0.8747 by using a portfolio of 10 solvers and is able to close the 90.38% of the gap between the SBS and VBS. 3S is however very close to SUNNY, and in particular the difference between the best performance of 3S (0.8651 with 9 solvers) and the peak performance of SUNNY is minimal (about 0.96%).

It is interesting to notice that, unlike the others, the OTS approaches have non monotonic performances when the portfolio sizes increases. This is particularly evident looking at their performance decrease when a portfolio of size 7 is used instead of one with just 6 solvers. This instability is obviously a bad property for a portfolio approach and it is probably due to the fact that the classifiers on which OTS approaches rely become inaccurate when they have to chose between too many



(a) Results considering all the approaches and the VBS.



(b) Results considering SBS, VBS, and the best two approaches.

Figure 5.2: Average Score.

candidate solvers (a similar behaviour was noticed also in the CSP evaluations of Chapter 4).

SATzilla lies in the middle between OTS approaches and the scheduling-based approaches 3S and SUNNY. The performance difference is probably due to the fact that SATzilla selects only one solver to run. Many COP solvers (especially the FD solvers that do not rely on lazy clause generation or MIP techniques) are indeed able to quickly find good sub-optimal solutions, even though they may fail to further improve later. Therefore, although at a first glance it may seem counterintuitive, the choice of scheduling more than one solver turns out to be effective also for COPs. In this way the risk of predicting a bad solver is reduced, while the overall quality of the solving process is often preserved.

Let us now discuss the `score` parametrization. We conjecture that varying the α and β parameters has not a big impact on the solvers ranking. Figure 5.3 reports the average score by setting the score parameters to $\alpha = 0$ and $\beta = 1$. We have chosen these bounds because 0 and 1 are the extremes of α and β parameters. The resulting function can be seen as a metric that only measures the quality of a solution, without discriminating whether the optimality is proven or not. This might make sense for applications in which it is very difficult to prove the optimality. There is however one major drawback: the $\texttt{score}_{0,1}$ metric does not discriminate between the absence of solutions (perhaps due to a buggy solver) and a solution of low quality (but still a sound solution).

As can be seen, the plot is rather similar to the one presented in Figure 5.2. The overall ranking of the solvers is still preserved. The performance difference between 3S and SUNNY becomes definitely negligible, but SUNNY still achieves the best score (0.9784 with nine solvers, while the peak performance of 3S is 0.9781). SATzilla remains in the middle between them and the OTS approaches, with a peak performance of 0.9154 (eleven solvers). Finally, RF and SMO reach the best value with six solvers (0.8949 and 0.8948 respectively) and still have a performance deterioration when adding more solvers. The SBS, which does not appears in the plot for the sake of readability, is pretty far away since its average $\texttt{score}_{0,1}$ is about

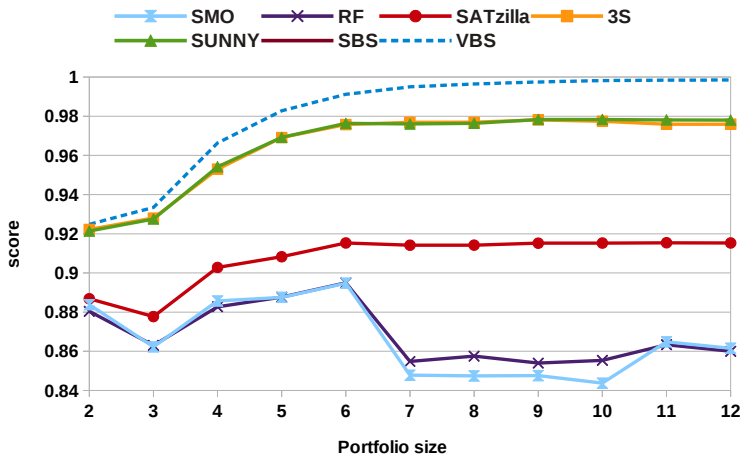


Figure 5.3: Average of the $\texttt{score}_{0,1}$ metric.

0.7181.

A systematic study of the sensitivity of all the portfolio approaches towards the α and β parameters of $\texttt{score}_{\alpha,\beta}$ is not considered here. However, according to these findings, we suppose that the solvers ranking does not change significantly when reasonable values of α and β are used. The correlation between `score` and $\texttt{score}_{0,1}$ values is also confirmed by the Pearson coefficient, which is very high: 0.89.

5.1.3.2 Optima Proven

Figure 5.4 shows (in percentage) the number of optima proven by the portfolio approaches, setting as additional baselines the performances of the SBS and the VBS. This is the equivalent of the PSI metric given in Def. 4.1, with the only difference that here we use the `proven` metric defined for COPs in Def. 5.1.

Looking at the plot, it is clear the demarcation between SUNNY and the other approaches. SUNNY appears to prove far more optima w.r.t. the other techniques, reaching the maximum of 55.48% with a portfolio of 10 solvers. We think that the performances of SUNNY are due to the fact that it properly schedules more than one solver. Moreover, it uses this schedule for the entire time window (on the contrary, 3S uses a static schedule only for 10% of the time window). Another interesting fact is that SUNNY mimics the behaviour of the VBS. Thus, SUNNY seems able to

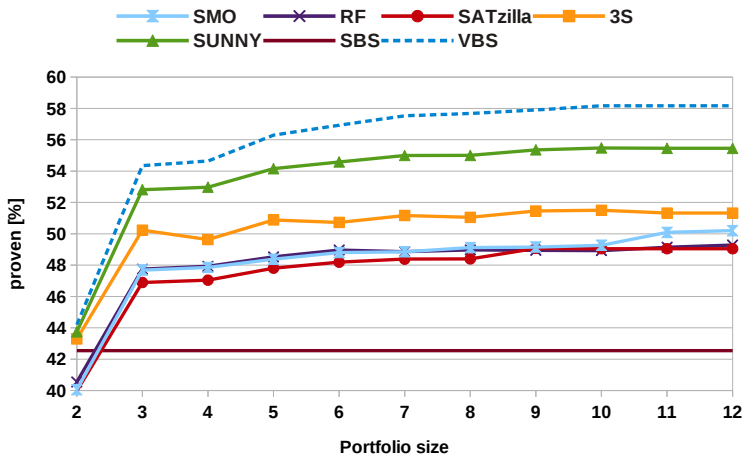


Figure 5.4: Percentage of Optima Proven.

properly exploit the addition of a new solver by taking advantage of its contribution in terms of `proven`.

The closest approach to SUNNY is 3S, which reaches a maximum of 51.51% with 10 solvers. The other approaches —that selects just one solver per instance— appear to be worse than SUNNY and 3S, and fairly close to each other. Moreover, it is evident that all the portfolio approaches greatly outperform the SBS. SUNNY in particular is able to close the 82.78% of the gap between the SBS and VBS.

It is interesting to notice that `proven` is actually not so dissimilar from `score`: the Pearson coefficient is about 0.72. A possible explanation is that the `proven` function is actually equivalent to the $\texttt{score}_{0,0}$ function.

5.1.3.3 Optimization Time

Figure 5.5 presents the average optimization times of the various approaches, setting as additional baselines the performances of the SBS and the VBS. It is easy to note that this metric is the equivalent of the the AST metric given in Def. 4.2 for CSPs. Clearly, here we use the `time` metric defined for COPs in Def. 5.2.

Even in this case, the results turn out to be related to those previously reported. The only remarkable difference concerns the 3S approach, that here does not perform as well as before. We think that this is due to the fact that 3S is a portfolio

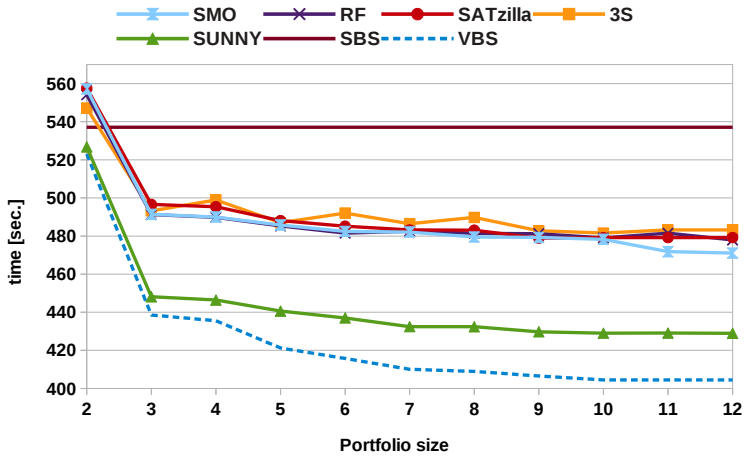


Figure 5.5: Average Optimization Time.

that schedules more than one solver and it does not explicitly employ heuristics to decide which solver has to be executed first. SUNNY instead does not suffer from this problem since it schedules the solvers according to their performances on the already known instances. However, the performances of 3S look very close to those of SATzilla and the OTS approaches.

Even in this case we can observe the good performance of SUNNY that is able to close the 81.55% of the gap between SBS and VBS reaching a peak performance of 428.95 seconds with a portfolio of 12 solvers. The second position is this time achieved by SMO, that with 12 solvers reaches a minimum of 471.05 seconds.

As one can expect, `time` is by definition strongly anti-correlated to `score` (the Pearson coefficient is -0.88) and however moderately related also to the `score` metric (the Pearson coefficient is -0.63).

5.1.3.4 MiniZinc Challenge Score

In this section we present the results obtained by considering the scoring metric of the MiniZinc Challenge, hereafter abbreviated as MZCS (MiniZinc Challenge Score).

Given the nature of MZCS (see Section 5.1.1) it makes no sense to introduce the VBS. In fact, due to the Borda comparisons, this addition would only cause a lowering of the scores of the other "real" solvers. Conversely, it is certainly reasonable to

consider the SBS against the portfolio solvers. In Figure 5.6 is depicted the average score achieved by every portfolio approach and the SBS.

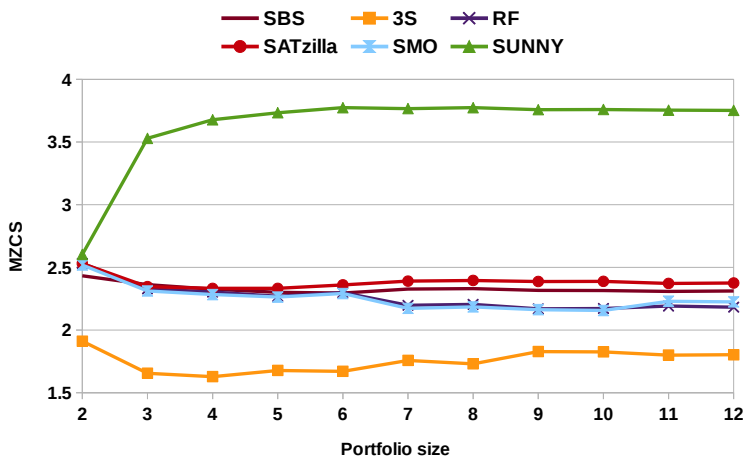


Figure 5.6: MiniZinc Challenge Score.

It interesting to note that the results are somehow different from what observed using other evaluation metrics. Even if SUNNY is still the best approach, we notice a remarkable improvement of SATzilla as a well as a significant worsening of 3S, that turns out to be always worse than the SBS. We think that this is due to the fact that SATzilla is able to select solvers that give better answers w.r.t. the OTS approaches (especially when optimality is not proven) and that, in case of indistinguishable answers, the scheduling-based approach of 3S is penalized by the Borda count used in the MiniZinc Challenge.

Note that, due to the pairwise comparisons between the solvers, the performance of the SBS is not constant. Indeed, its performance depends on the performance of the other solvers of the portfolio. The better a solver becomes, the more solvers it is able to beat, thus improving its score. Moreover, differently from `score`, here the score is not between 0 and 1 since MZCS is a value in the range $[0, m-1]$ where m is the number of the solvers involved in the Borda count. The peak performance is achieved by SUNNY that with 6 solvers has an average score of 3.77, meaning that on average it is able to beat almost 4 out of 5 competitors per instance. All the other approaches reach instead the peak performance with 2 solvers, in correspondence

with the worst SUNNY performance.

The different nature of the MZCS metric w.r.t. the other metrics is also underlined by the very low correlation. In particular, there is in practice no correlation with `proven` (0.31) and `time` (-0.07). More pronounced, but still rather low, the correlation between MZCS and `score`: about 0.53.

5.1.3.5 Summary

We tackled the problem of developing portfolio approaches for solving COPs. Firstly, we introduced a function `score` which takes into account the quality of the solvers solutions. Then, we used such a metric (and others) for simulating and comparing different portfolio techniques properly adapted to the COP world.

The results obtained clearly indicate that, even in the optimization setting, the portfolio approaches have remarkable better performances than a single solver. We observed that, when trying to prove optimality, the number of times a solver cannot complete the search is not negligible. Moreover, the optimization times have a heavy-tailed distribution typical of complete search methods [85]. Hence, a COP setting can be considered an ideal scenario to apply portfolio approaches and obtain statistically better solvers by exploiting existing ones [84].

We noticed that, even though at a first glance it can seem counterintuitive, the best performances were obtained by SUNNY. This strategy reduces the risk of choosing the wrong solver and, apparently, this is more important than performing part of the computation again, as could happen when two (or more) solvers are run on the same instance.

It is important to emphasize that the results reported in this section can be considered as a "lower bound". Indeed, as done in the experiments of Chapter 4, we did not actually execute each portfolio solver but we simulated its outcome by exploiting the information already computed. This is certainly useful for avoiding the enormous effort of running every portfolio solver on each fold/repetition. However, as formerly stressed, the estimations reported in this section just consider each

constituent solver individually, without taking into account the potential use of "co-operative strategies" among the constituent solvers. In the next section we see a possible way to leverage the collaboration between different COP solvers.

5.2 Sequential Time Splitting and Bound Communication

As seen so far, *scheduling* a subset of the constituent solvers of a given portfolio seems to be a good strategy (see for instance the performance of SUNNY, but also [113, 92, 16, 160, 151]) . Since often the solving time of hard combinatorial problems is either relatively short or very long, the scheduling approach naturally handles the heavy tailed nature of solving.

In particular, in a COP setting a solver can *transmit* useful bounds information to another one if they are scheduled sequentially. In a portfolio scenario, a partial solution found by a solver s_1 is indeed a token of knowledge that s_1 can pass to another solver s_2 in order to prune its search space and therefore possibly improve its solving process. In this section we thus address the problem of boosting optimization by exploiting the sequential cooperation of different COP solvers. To do so, we will introduce the notion of *solving behaviour* for taking into account the anytime performance of the solvers.

This section basically reports the work we presented in [12].

5.2.1 Solving Behaviour and Timesplit Solvers

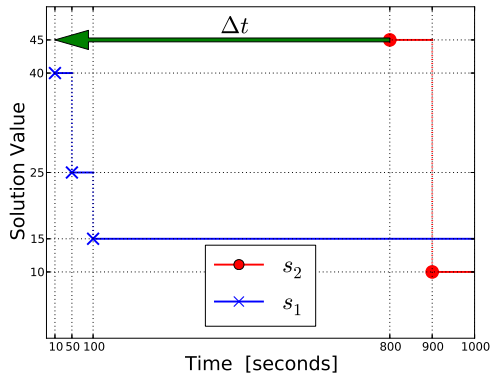
Let us fix a dataset $\mathbb{D}$ of COPs, a set $\mathbb{S}$ of COP solvers, and a solving timeout T. We wish to determine the best sequence of solvers in $\mathbb{S}$ to run on p and for how long to run each solver within $[0, T]$ in order obtain the best result for instance p. Ideally we aim to *improve the best solver* of $\mathbb{S}$ for the instance p. To do so, we introduce the concept of solving behaviour.

Definition 5.4 (Behaviour) *Let us fix a dataset $\mathbb{D}$ of COPs, a set $\mathbb{S}$ of COP solvers, and a solving timeout T. We define the* ***(solving) behaviour*** *$\mathcal{B}(s,p)$ of a solver $s \in \mathbb{S}$ applied to a problem $p \in \mathbb{D}$ over time $[0,T]$ as an ordered sequence of pairs $\mathcal{B}(s,p) := [(t_1,v_1),(t_2,v_2),\ldots,(t_n,v_n)]$ such that:*

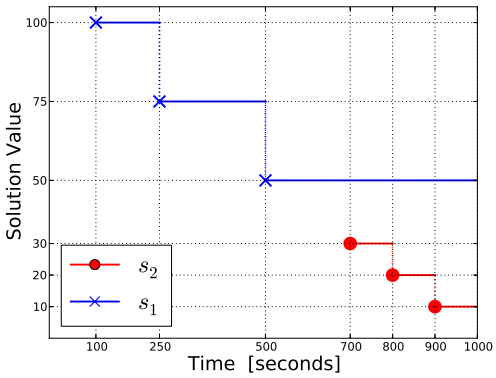
- *$\mathcal{B}(s,p) \subseteq [0,T) \times \mathbb{R}$;*
- *t_i is the time when s finds a solution ($i = 1,2,\ldots,n$);*
- *$v_i = \texttt{val}(s,p,t_i)$ is the objective value of such a solution ($i = 1,2,\ldots,n$).*

We can consider the pairs ordered so that $t_1 < t_2 < \cdots < t_n$ while $v_1 > v_2 > \cdots > v_n$ since we assume the solving process is monotonic.

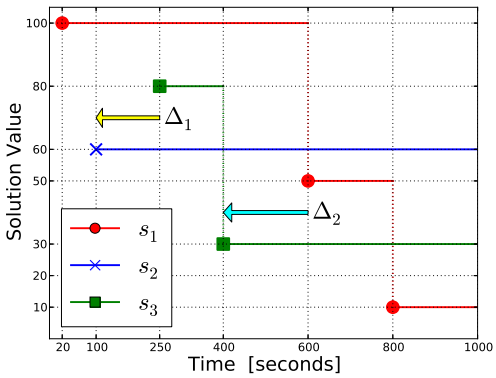
For example, consider the behaviours $\mathcal{B}(s_1,p) = [(10,40),(50,25),(100,15)]$ and $\mathcal{B}(s_2,p) = [(800,45),(900,10)]$ illustrated in Figure 5.7a with a timeout of $T = 1000$ seconds. The best value $v^* = 10$ is found by s_2 after 900 seconds, but it takes 800 seconds to find its first solution ($v = 45$). Meanwhile, s_1 finds a better value ($v = 40$) after just 10 seconds and even better values in just 100 seconds. So, the question is: what happens if we *"inject"* the upper bound 40 from s_1 to s_2? Considering that starting from $v = 45$ the solver s_2 is able to find v^* in 100 seconds (from 800 to 900), hopefully starting from any better (or equal) value $v' \leq 45$ the time needed by s_2 to find v^* is no more than 100 seconds. Note that from a graphical point of view what we would like to do is therefore to *"shift"* the curve of s_2 towards the left from $t = 800$ to 10, by exploiting the fact that after 10 seconds s_1 can suggest to s_2 the upper bound $v = 40$. The cooperation between s_1 and s_2 would thereby reduce by $\Delta t = 790$ seconds the time needed to find v^*, and moreover would allow us to exploit the remaining Δt seconds for finding better solutions or even proving the optimality of v^*. However, note that the *virtual* behaviour may not occur: it may be that s_2 calculates important information in the first 800 seconds required to find the solution $v^* = 10$, and therefore the injection of $v = 40$ could be useless (if not harmful).



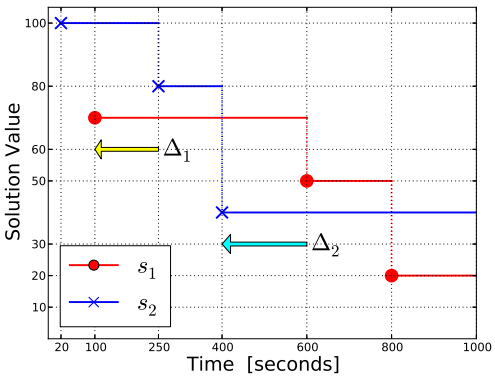
(a) $\sigma = [(s_1, 10), (s_2, 990)]$



(b) $\sigma = [(s_2, 1000)]$



(c) $\sigma = [(s_2, 100), (s_3, 150), (s_1, 750)]$



(d) $\sigma = [(s_1, 100), (s_2, 150), (s_3, 750)]$

Figure 5.7: Examples of solving behaviours and corresponding time splitting σ.

Definition 5.5 (Timesplit Solver) *Given a problem $p \in \mathbb{D}$ and a schedule of solvers $\sigma := [(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)] \subseteq \mathbb{S} \times [0, T)$ we define the corresponding* ***timesplit solver*** *as a particular solver that:*

(i) first, runs s_1 on p for t_1 seconds;

(ii) then, for $i = 1, 2, \ldots, k-1$, runs s_{i+1} on p for t_{i+1} seconds possibly exploiting the best solution found by the previous solver s_i in t_i seconds.

The ***base*** *of the timesplit solver σ is a timesplit solver $\underline{\sigma}$ where we omit the last solver in the schedule: $\underline{\sigma} := [(s_1, t_1), (s_2, t_2), \ldots, (s_{k-1}, t_{k-1})]$.*

We can see a timesplit solver as a "compound solver" that not only schedules the solvers $s_1, s_2, \ldots, s_m$ but also exploits their sequential cooperation. The base $\underline{\sigma}$ of a timesplit solver σ is itself a timesplit solver that ideally contributes to improve the last scheduled solver s_k. Note that, for simplicity, we will use the same notation to denote a timesplit solver and its corresponding schedule. As an example, in Figure 5.7a the ideal timesplit solver would be defined by $\sigma = [(s1, 10), (s2, 990)]$, while $\underline{\sigma} = [(s1, 10)]$. There are however cases in which the timesplit solver is actually a single solver, since the best solver is not virtually improvable by any other. This happens when every solution found by the best solver is also the best solution found so far (e.g., see Figure 5.7b). Moreover, there may be also cases in which splitting the time window in more than two slots (even alternating the same solvers) may ideally lead to better performances. Indeed, the "overall" best solver at the time edge T might no longer be the best one at a previous time $t < T$. For example, in Figure 5.7c the best solver at time $t \geq 800$ is s_1, at time $400 \leq t < 800$ is s_3 while for $t \leq 400$ is s_2; in Figure 5.7d the best solver is s_1 if $t < 400$ or $t \geq 800$, while for $400 \leq t < 800$ is s_2.

5.2.2 Splitting Selection and Evaluation

Once informally hypothesized the potential benefits of timesplit solvers, some questions naturally arise:

- First, which metric(s) is reasonable to formally define the "best solver"?
- Furthermore, how do we split the time window between solvers for determining the (virtually) best timesplit solver?
- Finally, to what extent do timesplit solvers act like the virtual timesplit solvers?

In order to answer these questions we fixed some proper metrics, defined a splitting algorithm and empirically evaluated the assumptions previously introduced.

5.2.2.1 Evaluation Metrics

In order to evaluate the performances of different COP solvers (and thus formally define the notion of best solver) we examine a number of metrics for grading a solver s on a problem p over a time limit T.

In addition to the `proven`, `time`, and `score` metric introduced above (see Def. 5.1, 5.2, 5.3 respectively) here we introduce a new metric able to estimate the *anytime* solver performance.

Definition 5.6 (`area`) *Let us fix a dataset of COPs $\mathbb{D}$, a set of COP solvers $\mathbb{S}$, and a solving timeout T. Given a problem $p \in \mathbb{D}$, let $W_p := \{\texttt{val}(s,p,t) \mid s \in \mathbb{S},\ t \in [0,T]\}$ be the set of all the solutions of p found by any solver at any time. If $\mathcal{B}(s,p) = [(t_1,v_1),(t_2,v_2),\ldots,(t_n,v_n)]$ is the behaviour of solver s on problem p, we define the **(solving) area** of s on p as a function $\texttt{area} : \mathbb{S} \times \mathbb{D} \to [0,T]$ such that:*

$$\texttt{area}(s,p) := t_1 + \sum_{i=1}^{n} \left(0.25 + 0.5 \cdot \frac{\texttt{val}(s,p,t_i) - \min W_p}{\max W_p - \min W_p}\right)(t_{i+1} - t_i)$$

where $t_{n+1} := \texttt{time}(s,p)$.

As the name implies, `area` is a normalized measure of the area under a solver behaviour. Actually the solving area could be seen as the cumulative sum of the anytime `score` of a solver in the solving time window. The main difference is that the `score` function scales the objective values in the range $[\min V_p, \max V_p]$, where $V_p =$

$\{\texttt{val}(s,p,T) \ : \ s \in \mathbb{S}\}$ is the set of the objective function values found by any solver s at the time limit T. Conversely, `area` uses instead the range $[\min W_p, \max W_p]$ for mapping all the solutions found by any solver at any time. Without this change, the partial solutions found at time $t < T$ might not be adequately addressed. Similarly to $\texttt{score}_{\alpha,\beta}$, even the `area` metric may be defined in terms of α, β parameters:

$$\texttt{area}_{\alpha,\beta}(s,p) = t_1 + \sum_{i=1}^{n} \left(\alpha + (\beta - \alpha) \cdot \frac{\texttt{val}(s,p,t_i) - \min W_p}{\max W_p - \min W_p} \right) (t_{i+1} - t_i)$$

However, as for the score, we assume $\alpha = 0.25$ and $\beta = 0.75$.

Example 5.3 *Consider the behaviours of the solvers s_1, s_2, s_3 on a minimization problem p shown in Example 5.1 on page 91. Precisely, the behaviours are:*

- $\mathcal{B}(s_1,p) = [(100,70),(200,60),(500,40)]$*;*
- $\mathcal{B}(s_2,p) = [(400,50)]$*;*
- $\mathcal{B}(s_3,p) = [(550,30),(650,20),(700,10)]$.

The optimization times are instead $\texttt{time}(s_1,p) = \texttt{time}(s_2,p) = T = 1000$ *while* $\texttt{time}(s_3,p) = 900$. *The set of the anytime values is* $W_p = \{10,20,30,40,50,60,70\}$. *The scaling of such values in* $[0.25, 0.75]$ *produces the corresponding normalized values* $\overline{W}_p = \{0.25 + 0.5 \cdot \frac{v-10}{60} : v \in W_p\} = \{0.25, 0.33, 0.42, 0.5, 0.58, 0.67, 0.75\}$. *Figure 5.8 depicts the "normalized" behaviours of such solvers, where instead of the original objective values of W_p we show the corresponding normalized values of $\overline{W}_p$.*

The area of s_1 *is* $\texttt{area}(s_1,p) = 100 + 0.75 \cdot (200-100) + 0.67 \cdot (500-200) + 0.5 \cdot (1000-500) = 626$. *The area of* s_2 *is* $\texttt{area}(s_2,p) = 400 + 0.58 \cdot (1000-400) = 748$ *while* $\texttt{area}(s_3,p) = 550 + 0.42 \cdot (650-550) + 0.33 \cdot (700-650) + 0.25 \cdot (900-700) = 658.5$. *Note that even if* s_3 *finds the optimal value* $v^* = 10$*, and proves its optimality, its area is worse than that of* s_1 *(even if the best value found by* s_1 *is 40, which is worse than the worst value found by* s_3*). This is because* `area` *rewards more a solver that quickly finds good solutions rather than one that slowly finds the best solutions (maybe by also proving the optimality).*

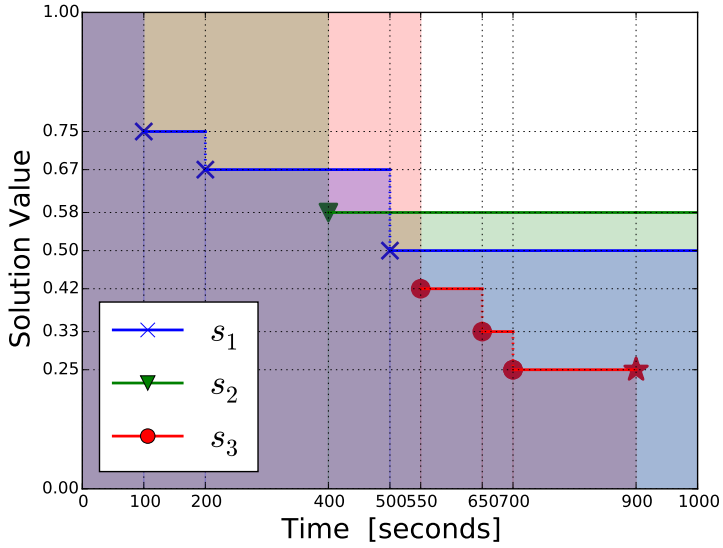


Figure 5.8: Solving area of s_1, s_2, s_3.

The solving area is similar to the *primal integral* [27] used for measuring impact of heuristics for MIP solvers, but differs since the primal integral assumes the optimal solution is known, while `area` also differentiates between finding and proving a solution optimal. The `area` metric folds in a number of measures the strength of an optimization algorithm: the quality of the best solution found, how quickly any solution is found, whether optimality is proven, and how quickly good solutions are found. Even though the ideal goal is to find the best objective value and hopefully proving its optimality, `area` allows us to discriminate much more between solvers, since we capture the trade-off between speed and solution quality. Two solvers which eventually reach the same best solution (without proving optimality) are indistinguishable with the other measures, but we would almost certainly prefer the solver that finds the solution(s) faster. Furthermore, consider two solvers that prove optimality at the same instant $t < T$: while both will have $\texttt{time} = t$, `area` will reward the solver in $[0, t]$ that finds better solutions faster.

Finally, we can now define the *best solver* of $\mathbb{S}$ for a given problem p as the solver $s \in \mathbb{S}$ which minimizes (w.r.t. the lexicographic ordering) the set of triples $(1 - \texttt{score}(s, p), \texttt{time}(s, p), \texttt{area}(s, p))$. Intuitively, the best solver is the one that

finds the best solution within the time limit T, breaking ties by using minimum optimization time first, and then minimum area (i.e., giving priority to the solvers that prove optimality in less time, or at least that quickly find sub-optimal solutions). Note that it is not unlikely that two different solvers s_1 and s_2 have the same value of `score` and `time`. For example, consider a problem p for which after T seconds s_1 and s_2 find the same objective value v without proving its optimality. By definition, $\texttt{score}(s_1, p) = \texttt{score}(s_2, p)$ and $\texttt{time}(s_1, p) = \texttt{time}(s_2, p) = T$. Hence, the best solver between s_1 and s_2 will be the one with lower `area`.

5.2.2.2 `TimeSplit` Algorithm

Our goal is now to find a suitable timesplit solver for instance p which can improve upon the best solver for p. The algorithm `TimeSplit` described with pseudo-code in Listing 5.1 encodes what was informally explained earlier (see Figure 5.7). Given as input a problem p, a portfolio $\Pi \subseteq \mathbb{S}$, and the timeout T, the basic idea of `TimeSplit` is to start from the behaviour of the best solver $s_2 \in \Pi$ for p and then examine other solvers behaviours looking for the maximum ideal "left shift" toward another solver $s_1 \in \Pi \setminus \{s_2\}$. Then, starting from s_1, this process is iteratively repeated until no other shift is found. The best solver of Π is assigned to s_2 via function `best_solver` in line 2, while line 3 sets the current schedule σ to $[(s_2, T)]$. In line 4 auxiliary variables are initialized: *tot_shift* keeps track of the sum of all the shifts identified, *max_shift* is the current maximum shift that s_2 can perform, *split_time* is the time instant from which s_2 will start its execution, while *split_solver* is the solver that has to be run before s_2 until *split_time* instant. The `while` loop enclosed in lines 5-18 is repeated until no more shifts are possible (i.e., $max\_shift = 0$). The three nested loops starting at lines 7-9 find two pairs (t_1, v_1) and (t_2, v_2) such that s_2 can virtually shift to another solver s_1, i.e., such that in the current solving window $[0, split\_time]$ we have that at time $t_1 < t_2$ solver s_1 finds a value v_1 better than or equal to v_2. If the actual shift $\Delta t = t_2 - t_1$ is greater than *max_shift*, in lines 11-13 the auxiliary variables are updated accordingly. In line 15, the allocated time of the current first solver of σ (i.e., s_2) is decreased by an amount of time $max\_shift + split\_time$ (note

that `first`(σ) is a reference to the first element of σ, while `snd` returns the second element of a pair, i.e. the allocated time in this case). This is because *split_time* seconds will be allocated to *split_solver* (line 16: `push_front` inserts an element on top of the list) while *max_shift* seconds corresponding to the ideal shift will be later donated to the 'overall' best solver of Π (i.e., the last solver of σ) via tot_shift variable.

```
1  TimeSplit(p, Π, T):
2     s_2 = best_solver(p, Π, T)
3     σ = [(s_2, T)]
4     tot_shift = 0  ;  max_shift = 1  ;  split_time = T  ;  split_solver = s_2
5     while max_shift > 0:
6        max_shift = 0
7        for (t_2, v_2) in {(t, v) ∈ B(s_2, p) | t ≤ split_time}:
8           for s_1 in Π \ {s_2}:
9              for (t_1, v_1) in {(t, v) ∈ B(s_1, p) | t < t_2 ∧ v ≤ v_2}:
10                if t_2 − t_1 > max_shift:
11                   max_shift = t_2 − t_1
12                   split_time = t_1
13                   split_solver = s_1
14       if max_shift > 0:
15          first(σ).snd −= max_shift + split_time
16          push_front(σ, (split_solver, split_time))
17          tot_shift += max_shift
18          s_2 = split_solver
19    last(σ).snd += tot_shift
20    return σ
```

Listing 5.1: `TimeSplit` Algorithm.

At this stage, the search for a new shift is restricted to the time interval $[0, split\_time]$ in which the new best solver s_2 will be *split_solver* (line 18). Once out of the `while` loop (no more shifts are possible) the total amount of all the shifts found is added to the best solver (line 19: `last`(σ) is a reference to the last element of σ) and the final schedule is finally returned in line 20.

5.2.2.3 `TimeSplit` Evaluation

In order to experimentally verify the correctness of our assumptions on the behaviour of timesplit solvers, we tested `TimeSplit` by considering a portfolio Π constructed from the solvers of the MiniZinc 1.6 suite (i.e., CPX, G12/FD, G12/LazyFD, and G12/MIP) with some additional solvers disparate in their nature, namely: Gecode [73] (CP solver), MinisatID [50] (SAT-based solver), Chuffed (CP solver with Lazy Clause Generation), and G12/Gurobi (MIP solver). We retrieved and filtered an initial dataset $\mathbb{D}$ of 4864 MiniZinc COPs from MiniZinc 1.6 benchmarks and the MiniZinc Challenges 2012/13 and then ran `TimeSplit` using a solving timeout of $T = 1800$ seconds. In particular, we ran and compared two versions of the algorithm: the original one and a variant (denoted `TS-2` in what follows) in which we imposed a maximum size of 2 solvers for each schedule σ. This is because splitting $[0, T]$ in too many slots can be counterproductive in practice: excessive fragmentation of the time window may produce time slots that are too short to be useful. Once executed these algorithms, in order to evaluate their significance we discarded all the "degenerate" instances for which the potential total shift was minimal (less than 5 seconds). We then ended up with a reduced dataset $\mathbb{D}' \subset \mathbb{D}$ of 596 instances. We ran timesplit solvers defined by the schedule returned by each algorithm on every instance of $\mathbb{D}'$. In addition, we added as a baseline the Virtual Best Solver (VBS). Finally, we evaluated and compared the average performance in terms of the above mentioned metrics: `score`, `proven`, `time`, `area`.

Table 5.2 shows the average results for each approach. As can be seen, the performances are rather close. On average, VBS is still the best solver if we focus on `score` metric (i.e., considering only the values found at the time limit T). Regarding `proven` and `time` metrics, we can observe a substantial equivalence: VBS is slightly better in terms of percentage of optima proven, while it is worse than `TimeSplit` and `TS-2` if we consider the average time to prove optimality. Conversely, looking at `area` the situation appears to be more clearly defined: on average, VBS is substantially worse than both `TimeSplit` and `TS-2`. This means that, even if the virtual behaviour does not always occur, often the time splitting we propose is able to find good partial

solutions more quickly than the best solver of Π. Focusing just on the two versions of `TimeSplit`, we can also note that these are substantially equivalent: this confirms the hypothesis that limiting the algorithm to schedule only two solvers is a reasonable choice (`TS-2` seems slightly better than `TimeSplit` on average). Indeed, among all the instances of $\mathbb{D}'$, only for 53 of them `TimeSplit` has produced a schedule with more than two solvers. Table 5.3 shows instead how many times the approach on the i-th row is better than the one on the j-th column. In this case we can note that `TimeSplit` and `TS-2` perform better than VBS: indeed, in the cases in which the score is the same for both the approaches, often the timesplit solvers take less time to find a (optimal) solution.

	`score`	`proven`	`time`	`area`
VBS	**82.40%**	**34.73%**	1298.67	478.05
`TimeSplit`	80.49%	33.67%	1263.74	347.91
`TS-2`	80.60%	33.89%	**1259.98**	**343.97**

Table 5.2: Average performances.

	VBS	`TimeSplit`	`TS-2`
VBS	—	222	232
`TimeSplit`	**373**	—	40
`TS-2`	364	13	—

Table 5.3: Pairwise Comparisons.

Note that for 375 problems (62.92% of $\mathbb{D}'$) *at least one* between `TimeSplit` and `TS-2` is better than the VBS, i.e., the best solver of $\{\texttt{TimeSplit}, \texttt{TS-2}, \text{VBS}\}$ is `TimeSplit` or `TS-2`. Let $\mathbb{D}^*$ be the set of such instances, and considering the base $\underline{\sigma(p)}$ of each schedule $\sigma(p)$ returned by the best approach between `TimeSplit` and `TS-2` for each instance of $p \in \mathbb{D}^*$, we noticed an interesting fact: the time allocated to $\underline{\sigma(p)}$ is usually pretty low. Figure 5.9 reports the distribution of such each times. As can be seen, almost all the times are concentrated in the lower part of the graph:

even if the maximum value is 1363 seconds, the mean is less than a minute (54.18 seconds to be precise) while the median value is significantly lower (9 seconds).

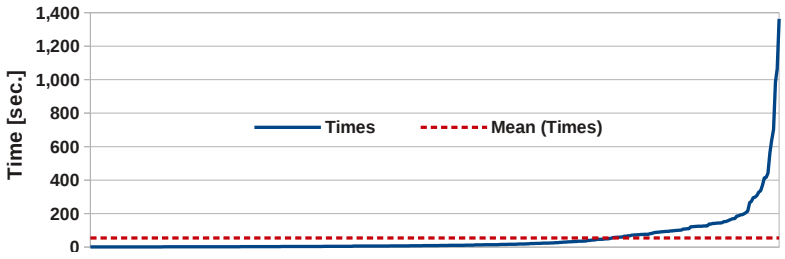


Figure 5.9: Times allocated to $\underline{\sigma(p)}$ for each $p \in \mathbb{D}^*$.

Figure 5.10 shows an example of the benefits of timesplit solvers on an instance of the Resource-Constrained Project Scheduling Problem (RCPSP) [34] taken from the MiniZinc suite.[7] The timesplit solver runs the schedule [(CPX, 1), (MinisatID, 1799)] since after just one second CPX is able to find the value 101. The point is that CPX never improves that value in the remaining 1799 seconds. Conversely, MinisatID is slower in finding good solutions (it finds 101 at time 799) but in the end it finds a better value than CPX (i.e., 100 at time 844). From Figure 5.10 we can see that —by receiving the bound 101 from CPX after 1 second— MinisatID is able to find the value 100 in just 173 seconds. Moreover, it finds the value 99 at time 191 and at time 217 it proves that 98 is an optimal value.

5.2.3 Timesplit Portfolio Solvers

The results of Section 5.2.2.3 show that in a non-negligible number of cases the benefits of using a timesplit solver are tangible. Unfortunately, in such experiments for every instance we already knew the corresponding runtimes of each solver of the portfolio. What we want is instead to *predict* and run the best timesplit solver for a new unseen instance. Regrettably, given runtime prediction of a solver is a non-trivial task, predicting the detailed solver behaviour on a new test instance is

[7] Precisely, the model is in `minizinc-1.6/benchmarks/rcpsp/rcpsp.mzn` while the corresponding data is in `minizinc-1.6/benchmarks/rcpsp/data_psplib/j120/J120_9_6.dzn`

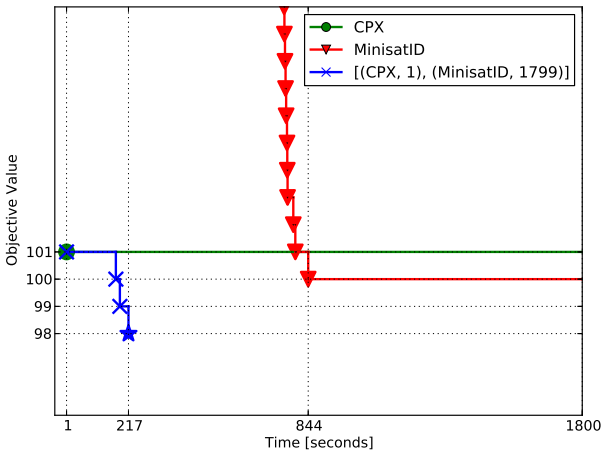


Figure 5.10: Splitting benefits on a RCPSP instance.

even harder. Indeed, in our case we can not simply limit ourselves to guess the best solver for a new instance, but we should instead predict a suitable timesplit solver $[(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)]$. Moreover, even if in most cases the `TimeSplit` algorithm works pretty well, on the others we noticed a considerable number of instances for which this algorithm is ineffective (or even harmful). Therefore, a successful strategy should be able not only to predict a proper timesplit solver, but also to distinguish between the instances for which the timesplit is actually useful and those where it is counterproductive. Furthermore, another interesting observation that has emerged from the results of Section 5.2.2.3 is that often for the "significant" timesplit solvers is sufficient to run the base of the schedule for a relatively low number of seconds in order to allow an effective improvement of the best solver.

On the basis of these observations and motivations, we propose a generic and hybrid framework that we called *Timesplit Portfolio Solver*.

Definition 5.7 (Timesplit Portfolio Solver) *Let $p \in \mathbb{D}$ be a COP and T a solving timeout. A* ***Timesplit Portfolio Solver*** *(`TPS`) is a timesplit solver defined as:*

$$\texttt{TPS}(p) := [(\mathcal{S}, C), (\mathcal{D}(p), T - C)]$$

where:

- $(\mathcal{S}, C)$ *is a* ***static*** *timesplit solver, pre-computed off-line, that will run for the first* $C < T$ *seconds;*
- $(\mathcal{D}(p), T - C)$ *is a* ***dynamic*** *timesplit solver, computed on-line by a given prediction algorithm* $\mathcal{D}(p)$*, that will run for the remaining* $T - C$ *seconds.*

The underlying idea of `TPS` is to exploit for the first C seconds a fixed schedule calculated *a priori*, whose purpose is to produce as many good sub-optimal solutions as possible. If after C seconds the optimality is still not proven, in the remaining $T-C$ seconds the algorithm $\mathcal{D}(p)$ tries to predict which is the best (timesplit) solver for p that will be executed taking advantage of any upper bound provided by $\mathcal{S}$.

Since `TPS` is a general model that can be arbitrarily specialized, in the rest of this section we explain in more detail what choices we made and what algorithms we used to define and evaluate (variants of) `TPS`.

5.2.3.1 Static Splitting

Drawing inspiration from what was done in [113] for SAT problems, we decided to compute a static schedule of solvers according to the outcomes of `TimeSplit` on a given set of training instances. While in [113] the authors solve a Resource Constrained Set Covering Problem (RCSCP) in order to get a schedule that maximizes the number of training instances that can be solved within a time limit of $C = T/10$ seconds, in our case the objective is different. What we would like is indeed to compute a schedule that may act as a good base for the solver(s) who will be executed in the remaining $T - C$ seconds. To do this, we first identify by means of `TimeSplit` algorithm the set $\mathbb{D}^*$ of all the training instances for which a timesplit solver outperforms the VBS.

Let $\sigma(p) = [(s_{p,1}, t_{p,1}), (s_{p,2}, t_{p,2}), \ldots, (s_{p,k}, t_{p,k})]$ be the schedule returned by the `TimeSplit` algorithm on each $p \in \mathbb{D}^*$. We look for a schedule $\mathcal{S}$ that maximizes the number of time slots $t_{p,i} \in \underline{\sigma(p)}$ for $i = 1, 2, \ldots, k-1$ that are *covered*, that is the portfolio solver allocates at least $t_{p,i}$ seconds to solver $s_{p,i}$. Again, note that we

consider the base $\underline{\sigma(p)}$ instead of $\sigma(p)$ since at this stage we are not interested in choosing the best solver: we want to determine an effective timesplit solver able to quickly find suitable sub-optimal solutions. However, a nice side-effect of this approach is that it also may be able to solve quickly those instances that are extremely difficult for some solvers but very easy for others.

For each $p \in \mathbb{D}^*$, we define $\nabla_p \subseteq \mathbb{S} \times [0, C]$ as the set:

$$\nabla_p = \{(s_{p,i}, t) \mid (s_{p,i}, t_{p,i}) \in \underline{\sigma(p)},\ t_{p,i} \leq t \leq C\}$$

of all the pairs $(s_{p,i}, t)$ that *cover* the time slot $t_{p,i}$ within C seconds. Named

$$\Pi^* = \bigcup_{p \in \mathbb{D}^*} \{s \in \Pi : (s, t) \in \nabla_p\}$$

the set of the solvers of a portfolio $\Pi \subseteq \mathbb{S}$ that appear in at least a ∇_p, and fixed $C = T/10$, we solve the following Set Covering problem:

$$\begin{aligned}
\min \left[(C+1) \sum_{p \in \mathbb{D}^*} y_p + \sum_{s \in \Pi^*} \sum_{t \in [0,C]} t\, x_{s,t} \right] \qquad & s.t. \\
y_p + \sum_{(s,t) \in \nabla_p} x_{s,t} \geq 1 \qquad & \forall p \in \mathbb{D}^* \\
\sum_{t \in [0,C]} x_{s,t} \leq 1 \qquad & \forall s \in \Pi^* \\
\sum_{s \in \Pi^*} \sum_{t \in [0,C]} t\, x_{s,t} \leq C & \\
y_p,\ x_{s,t} \in \{0, 1\} \qquad & \forall p \in \mathbb{D}^*,\ \forall s \in \Pi^*,\ \forall t \in [0, C]
\end{aligned}$$

For each pair (s, t) there is a binary variable $x_{s,t}$ that will be equal to one if and only if in $\mathcal{S}$ the solver s will run for t seconds. For each problem p, the binary variable y_p will be one if and only if $\mathcal{S}$ cannot cover any time slot of $\underline{\sigma(p)}$. Constraint $y_p + \sum x_{s,t} \geq 1$ imposes that instance p is covered (possibly setting $y_p = 1$ in the worst case) while $\sum t\, x_{s,t} \leq C$ ensures that $\mathcal{S}$ will not exceed the time limit C. The objective is thus to minimize the number of uncovered instances first (by means of $C + 1$ coefficient for each y_p), and the total time of $\mathcal{S}$ then (using the t coefficient for each $x_{s,t}$).

Note that the solution of the problem defines an allocation $\xi = \{(s,t) : x_{s,t} = 1\}$ and not actually a schedule: we still have to define the execution order of the solvers. Since the interaction between different solvers is not easily predictable, and neither generalizable, we decided to use a simple and reasonable heuristic: we get the schedule $\mathcal{S}$ by sorting each $(s,t) \in \xi$ by increasing allocated time t.

5.2.3.2 Dynamic Splitting

Once defined the static part of `TPS`, we want to determine an algorithm $\mathcal{D}(p)$ able to predict for a new unseen instance p a proper (timesplit) solver to run for $T - C$ seconds after $[(\mathcal{S}, C)]$. Inspired by the results already presented in Section 4.2.3 and 5.1.3 we made use of the SUNNY algorithm.

The reasons behind the choice of SUNNY are essentially two. First, even if originally designed for CSP portfolios, SUNNY turns out to perform well also on COPs. Second, SUNNY is not limited to predict a single solver but selects instead a schedule of solvers: in other terms, it implicitly returns a timesplit solver (e.g., see Example 5.2).

5.2.4 Empirical Evaluation

In order to measure the performances of the `TPS` instance described in Sections 5.2.3.1 and 5.2.3.2, in the following referred to as `sunny-tps`, we considered a solving timeout of $T = 1800$ seconds, a threshold of $C = T/10 = 180$ seconds for the static schedule, the portfolio $\Pi = \{$Chuffed, CPX, G12/FD, G12/LazyFD, G12/Gurobi, G12/MIP, Gecode, MinisatID$\}$ and the dataset $\mathbb{D}$ of 4864 MiniZinc instances mentioned in Section 5.2.2.3.

We evaluated all the approaches by using a 10-fold cross validation [15]. $\mathbb{D}$ was randomly partitioned in 10 disjoint folds $\mathbb{D}_1, \mathbb{D}_2, \ldots, \mathbb{D}_{10}$ treating in turn one fold $\mathbb{D}_i$ as the test set TS_i ($i = 1, 2, \ldots, 10$) and the union of the remaining folds $\bigcup_{j \neq i} \mathbb{D}_j$ as the training set TR_i. For each training set TR_i we then computed a corresponding static schedule $(\mathcal{S}_i, 180)$ as explained in Section 5.2.3.1, and for every instance $p \in$

TS_i we computed and executed the timesplit solver $[(\mathcal{S}_i, 180), (\mathcal{D}_i(p), 1620)]$ where $\mathcal{D}_i(p)$ is the schedule returned by SUNNY algorithm for problem p using a reduced solving window of $T - C = 1620$ seconds.[8] Contrary to what has been done in the previous experiments (see Sections 4.1.2, 4.2.3, and 5.1.3) we did not use a 5-repeated 5-fold cross validation. This was essentially done for reducing the effort of running every approach for $4864 \cdot 5 = 24320$ times. Indeed, it is worth nothing that while in Sections 4.1, 4.2, 5.1 the portfolio evaluation was based on *simulations* of the approaches according to the already computed behaviours of every solver of Π on every instance of $\mathbb{D}$, in this work all the approaches have been *actually* run and evaluated. Indeed, as shown also in Section 5.2.2.3, in this case we can not make use of simulations since the side effects of bounds communication are unpredictable in advance.

For computing $\mathcal{D}_i(p)$ the SUNNY algorithm has to retrieve the k instances of TR_i closest to p. In order to do so, a proper set of *features* has to be extracted from p (and each instance of TR_i). Instead of using the whole set of 155 features extracted by the `mzn2feat` tool described in 4.2.2.3) we decided to select a proper subset of them by exploiting the new extractor `mzn2feat-1.0`.[9] This tool is a new version of `mzn2feat` designed to be more portable, light-weight, flexible, and independent from the particular machine on which it is run as well as from the specific global redefinitions of a given solver. Indeed, `mzn2feat-1.0` does not compute features based on graph measures (since this process could be very time/space consuming), solver specific features (like global constraint redefinitions) and dynamic features (to decouple the extractor from a particular solver and from the given machine on which it is executed). In more detail, `mzn2feat-1.0` extracts in total 95 features. The variables (27), domains (18), constraints (27), and solving (11) features are exactly the same of `mzn2feat` (see Table 4.7). The objective features (8) are the 12 objective features of `mzn2feat` except the 4 features that involve graph measures. The global

[8] The time needed to solve the Set Covering problem of every $\mathcal{S}_i$ does not contribute to the total solving time, since this step is performed offline. However, in these experiments each $\mathcal{S}_i$ has been computed in a matter of seconds.

[9] Available at `https://github.com/jacopoMauro/mzn2feat`

constraints features are just 4 and no longer bound to the Gecode solver, namely: the number of global constraints n, the number of different global constraints m, the ratio m/n and the ratio n/c where c is the total number of constraints of the problem. We finally removed all the constant features and scaled them in $[-1, 1]$, obtaining thus a reduced set of 88 features.

As in Section 5.2.1, we evaluated the average performance of `sunny-tps` in terms of `score`, `proven`, `time`, `area` by varying the neighbourhood size k in $\{10, 15, 20\}$. Finally, we compared `sunny-tps` vs. the following approaches:

- SBS: the overall Single Best Solver of Π according to the given metric;
- VBS: the Virtual Best Solver of Π defined as in Section 5.2.2.3;
- `sunny-ori`: the original SUNNY algorithm evaluated in Section 5.1.3. It is in practice a portfolio solver in which the selected solvers are executed independently (i.e., without any bounds communication) in the whole time window $[0, T]$ without any static schedule;
- `sunny-com`: is a portfolio solver that acts basically as `sunny-ori`, with the only exception that solvers execution is not independent: the best value found by a solver within its time slot is subsequently exploited by the following solver of the schedule. In other terms `sunny-com` is a `TPS` that does not exploit the "warm start" provided by the static schedule $[(\mathcal{S}, C)]$, but only executes the dynamic schedule $[(\mathcal{D}(p), T)]$.

Note that for evaluating the portfolio solvers we did not consider only the set $\mathbb{S}$ of the single solvers available. Instead, we extended $\mathbb{S}$ to a superset $\overline{\mathbb{S}} = \mathbb{S} \cup \{$`sunny-com`, `sunny-ori`, `sunny-tps`$\}$ containing also the SUNNY variants. We did this because the bounds communication may allow to outperform the best solver of $\mathbb{S}$ for a given problem. In this case, the set of values originally considered for scaling the objective values for the `proven` and `area` metrics may be altered thus leading to inconsistent measurements. For instance, in the example in Figure 5.10 we must

include also the values found by the timesplit solver for having a fair comparison (no single solver is indeed able to find the values 98 and 99).[10]

5.2.4.1 Test Results

The average `score` results (in percent) by varying the k parameter are reported in Figure 5.11. The plot shows a clear pattern: SBS, `sunny-ori`, `sunny-com`, `sunny-tps`, and VBS are respectively sorted by increasing score for every value of k. In general, we can see a rather sharp separation between the various approaches: this witnesses the effectiveness of bounds communication for reaching a better score or, in other terms, for improving the objective value (possibly proving its optimality). For example, the percentage difference between `sunny-ori` and `sunny-com` ranges between 2.83% and 3.45%. Furthermore, running the static schedule for the first 180 seconds (and therefore shrinking the dynamic schedule of `sunny-com` in the remaining 1620 seconds) seems to be advantageous: `sunny-tps` is always better than SBS, `sunny-ori`, and `sunny-com`. The peak performance (86.91%) is reached with $k = 15$, but the difference with $k = 10$ and 20 is minimal (0.73% and 0.59% respectively). Considering $k = 15$, `sunny-tps` has an average score higher than SBS by 10.55%, and lower than VBS by 6.9%. Moreover, in 82 cases (1.69% of $\mathbb{D}$) it scores better than VBS.

When considering the `proven` metric (Figure 5.12) the performance difference between the different SUNNY approaches is not so pronounced. Indeed, `sunny-ori`, `sunny-com`, and `sunny-tps` are pretty close: for every k, the percentage difference between the worst and the best SUNNY approach ranges between 0.45% and 1.13%. In this case we can say that the remarkable difference in performance between the portfolio solvers and the SBS is mainly due to the SUNNY algorithm rather than the bounds communication. In other words, passing the bound is not so effective if we just focus on proving optimality. A possible explanation is that communicating an

[10] Technically `sunny-ori` can't outperform any single solver since it does not use any bound communication, and could be therefore be excluded from $\overline{\mathbb{S}}$. However, we realized that sometimes the behaviour of a solver is not purely deterministic.

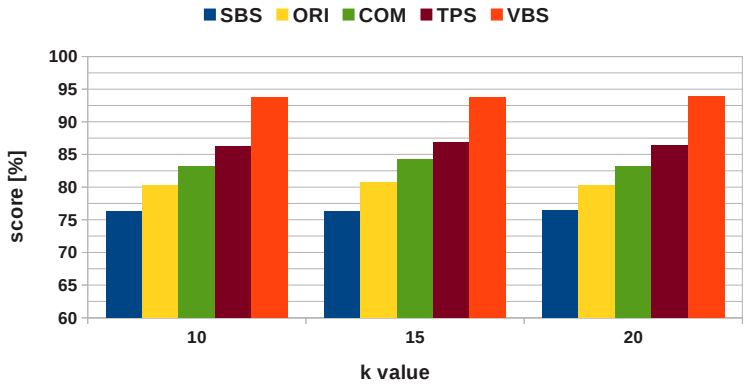


Figure 5.11: `score` results (in percent).

upper bound can be useful to find a better solution (see Figure 5.11) but ineffective when it comes to prove optimality. In these cases probably the time needed by a solver to compute information for completing the search process can not be offset by the mere knowledge of an objective bound. Nonetheless, the plot shows how the "warm start" provided by the static schedule is helpful: in fact, the performance of `sunny-tps` is always better than the other approaches. The peak performance ($k = 15$) is 72.06%, about 10.36% more than SBS and only 3.74% less than VBS. For 27 instances (0.56% of $\mathbb{D}$) `sunny-tps` is able to prove the optimality while VBS is not.

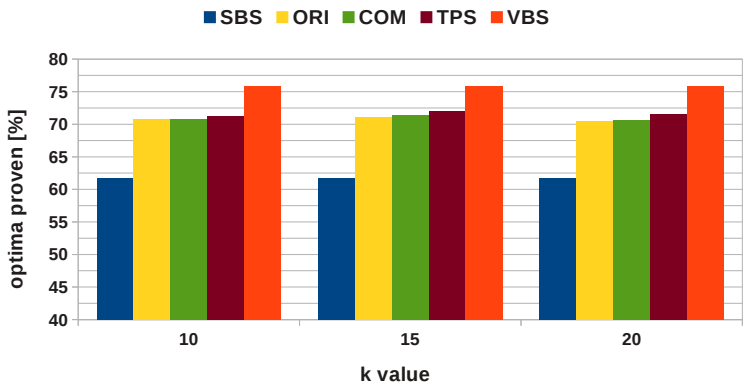


Figure 5.12: `proven` results (in percent).

Let us now focus on optimization time. In Figure 5.13 we see, in contrast to all the `score` and `proven` results that appear to be pretty robust by varying k, a slight discrepancy between $k = 10$ and $k > 10$. This delay time in proving optimality is due to the scheduling order of the constituent solvers. However, for $k = 15$ the results improve and for $k = 20$ are substantially the same. The peak performance is achieved with $k = 15$ (272.61 seconds), 105.07 less than SBS and 145.9 more than VBS; in 53 cases (1.09% of $\mathbb{D}$) `sunny-tps` is able to prove the optimality in less time than VBS.

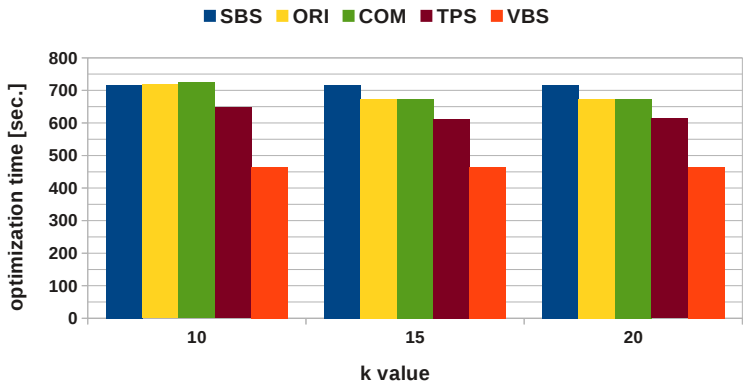


Figure 5.13: `time` results (in seconds).

The `area` results depicted in Figure 5.14 clearly show the benefits of bounds communication. First, note that `sunny-ori` is always worse than SBS: this is because each solver scheduled by `sunny-ori` is executed independently, and therefore for every solver the search is always (re-)started from scratch without exploiting previously found solutions. `sunny-com` significantly improves `sunny-ori`, even if its average area is very close to SBS (even worse for $k = 10$). The fixed schedule run by `sunny-tps` often allows one to quickly find partial solutions, and thus to noticeably outperform both `sunny-ori` and `sunny-com`. Like the `time` metric, the average area is not so close to VBS (the peak performance, with $k = 15$, is 272.61 seconds: 132.77 seconds more than VBS, and 114.9 less than SBS), but `sunny-tps` outperforms VBS in 110 cases (2.26% of $\mathbb{D}$).

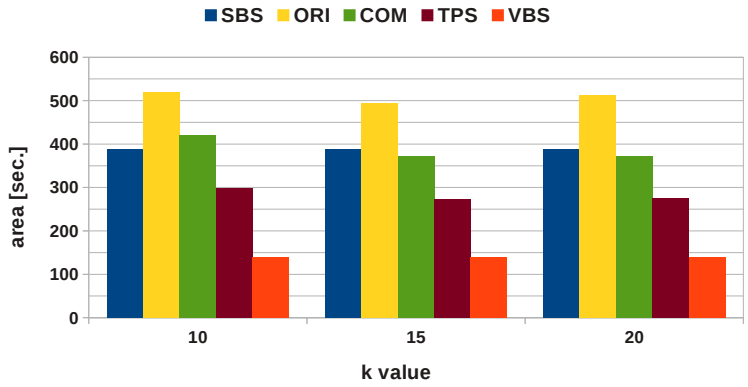


Figure 5.14: `area` results (in seconds).

5.2.4.2 Summary

In this section we addressed the problem of boosting optimization by exploiting the sequential cooperation of different COP solvers. A related work is [37], in which algorithm control techniques are used to share bounds information between the scheduled solvers without, however, explicitly rely on the solvers behaviours (as in the technical definition we gave in Def. 5.4). In [133] the authors provide a generic approach to knowledge sharing, which is suitable for SAT solvers but is less likely to be useful when solvers are very disparate in nature.

Exploiting the fact that finding good solutions early can significantly improve optimization solvers, we first provided a proper `TimeSplit` algorithm that relies on the behaviour of different solvers on an instance for determining a good time-split solver for this instance (i.e., ideally able to outperform the best solver of a portfolio). The results show that on average the actual timesplit solver does perform similarly to (and sometimes even better than) the Virtual Best Solver of the portfolio. We therefore exploited the results of `TimeSplit` in order to define the Timesplit Portfolio Solver (`TPS`), a generic and hybrid framework that combines a static schedule (computed off-line and run for a limited time) as well as a dynamic schedule (computed on-line by means of a proper prediction algorithm and run in the remaining time) for solving a new unseen instance by exploiting the bounds

communication between the scheduled solvers. In particular, on the one hand, we determined the static schedule by solving a Set Covering problem according to the results of `TimeSplit` on a set of training instances, and, on the other, we defined the dynamic selection by exploiting the SUNNY algorithm. Empirical results have shown that this idea can be beneficial and sometimes even able to outperform the Virtual Best Solver according to different metrics that we used (i.e., `score`, `proven`, `time`, and especially the new metric `area`) in order to evaluate the performance of different (portfolio) solvers.

We see this work a cornerstone for portfolio approaches to solving Constraint Optimization Problems. Future investigations in this regard may cover the construction of *meta-portfolios*, i.e., portfolio solvers consisting of other portfolio solvers. In a way, we can see a timesplit solver $\texttt{TPS}(p) = [(\mathcal{S}, C), (\mathcal{D}(p), T - C)]$ as a kind of meta-portfolio: it first runs the portfolio solver $(\mathcal{S}, C)$, and then the portfolio solver $(\mathcal{D}(p), T - C)$.

In the next chapter we show an effective tool exploiting (not only) these techniques for solving hard combinatorial problems.

Chapter 6

sunny-cp: a CP Portfolio Solver

"Manus Multae, Cor Unum"[1]

As previously stressed portfolio solvers are rarely used in practice, especially outside the walls of solving competitions. In Chapters 4 and 5 we showed several evaluations of different portfolio approaches for solving CSPs and COPs. In particular, we proposed a number of tools —built on top of SUNNY algorithm— for solving both CSPs (see `sunny-csp` in Section 4.2.3.3) and COPs (see `sunny-ori`, `sunny-com`, and `sunny-tps` in Section 5.2.4). Now, we merge these two lines of research by proposing `sunny-cp`: a new tool aimed at solving a generic CP problem, regardless of whether it is a CSP or a COP.

The aim of `sunny-cp` is to provide a flexible, configurable, and usable CP portfolio solver that can be set up and executed just like a regular individual CP solver. To the best of our knowledge, `sunny-cp` is currently the only sequential portfolio solver able to solve generic CP problems. Indeed, it was the only portfolio entrant in the MZC 2014 [139]. In this chapter we show the architecture of `sunny-cp` as well as an evaluation of its performance in the MZC 2014. As it is shown later, despite the MZC is not (yet) the ideal scenario for a portfolio solver, `sunny-cp` has proved to be competitive even in this setting.

This chapter reports an extended version of an our paper accepted in [11].

[1] "Many hands, one heart".

6.1 Architecture

In this section we illustrate the architecture of `sunny-cp`. We initially set up the portfolio of `sunny-cp` with the solvers introduced in Section 5.2.2.3, namely: Chuffed, CPX, G12/CBC, G12/FD, G12/Gurobi, G12/LazyFD, Gecode, and MinisatID. Clearly, it would have been possible to add a number of other solvers. However, from the experimental investigations reported in previous chapters it turned out that using too large portfolios may be ineffective or sometimes even harmful. We therefore decided to use just the eight solvers mentioned above, while still providing the opportunity to arbitrarily change the portfolio composition.

Figure 6.1 summarizes the step-by-step execution flow of the framework from the input CP problem to the final output outcome. The input of `sunny-cp` consists of the problem instance p to be solved, the size k of the neighbourhood used by the underlying k-NN algorithm, the solving timeout T, and the solver B of the portfolio to be used as backup solver. Despite the portfolio described above is fixed, it is still possible for the end user to select only a subset of its solvers. In addition, as described below, the user can also specify the knowledge base to use for the solver(s) selection.

The first step —that does not discriminate between CSPs and COPs— concerns the features extraction. Given a CP problem p in input, this process identifies the *feature vector* of p. By using the `mzn2feat-1.0` extractor mentioned in Section 5.2.4 we extract a vector $FV = \langle f_1, f_2, \ldots, f_d \rangle$ of $d = 95$ features.

At step 2, the execution flow branches: if p is a CSP instance, `sunny-cp` uses a knowledge base KB_{CSP}; otherwise, the knowledge base KB_{COP} is selected. Basically, a knowledge base can be seen as a map that associates to each CP problem a body of information relevant for the resolution process. Every CP problem of the knowledge base belongs to a training set of already known instances, and for each CP problem the relevant information are essentially two: its feature vector and the solving behaviour of each constituent solver on it. `sunny-cp` already comes with default knowledge bases, constructed by collecting two different datasets $\mathbb{D}_{CSP}$ and

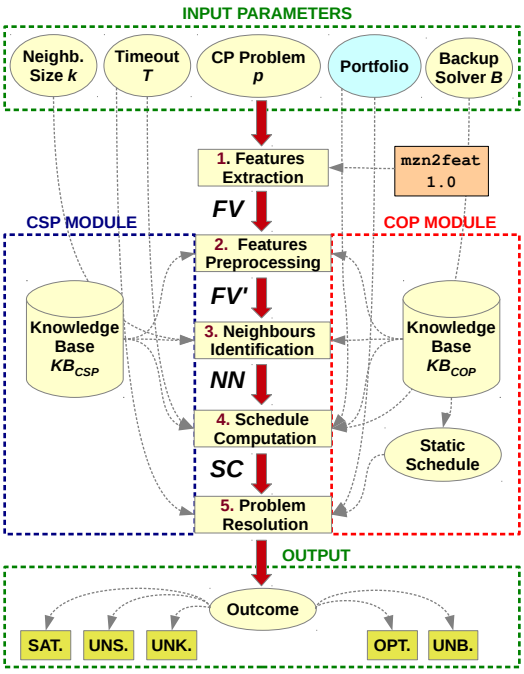


Figure 6.1: `sunny-cp` architecture.

$\mathbb{D}_{COP}$ of CSP and COP instances retrieved from the CP instances of the MiniZinc 1.6 benchmarks, the MZCs 2012/13, and the International CSP Solver Competitions. More in detail, $\mathbb{D}_{CSP}$ (resp. $\mathbb{D}_{COP}$) contains 5524 (resp. 4864) instances, to each of which is associated a feature vector of 78 (resp. 88) normalized features and the performances of each solver of the portfolio on it. The feature vectors of KB_{CSP} and KB_{COP} differ in size since from the original 95 features extracted by `mzn2feat-1.0` we removed all the constant features: as a consequence, the 10 features concerning the objective function has been removed from the CSPs feature vectors. Moreover, since the performances of the k-NN algorithm can be drastically degraded by the presence of noisy or irrelevant features [6], all the features values of KB_{CSP} and KB_{COP} are scaled in the range $[-1, 1]$. However, note that `sunny-cp` also allows to build and use other knowledge bases given as input parameters.

The original feature vector FV of the instance p is then normalized in step 2 by exploiting the information of the corresponding knowledge base. The resulting

normalized vector $FV' = \langle f'_1, f'_2, \ldots, f'_{d'} \rangle$ is therefore a feature vector in which every $f'_i \in [-1, 1]$ and $d' < d$ (in particular, as said above, $d' = 78$ if p is a CSP while $d' = 88$ if p is a COP).

The feature vector FV' is then used in step 3 to identify the neighbourhood NN of p, i.e, the k instances $p_1, p_2, \ldots, p_k$ of the selected knowledge base for which the corresponding feature vectors $FV'_1, FV'_2, \ldots, FV'_k$ minimize the Euclidean distance from FV'. The neighbourhood size k is an input parameter which is set to $k = 70$ unless otherwise specified. We chose this default value since it is close to the square root of the default knowledge bases size. Indeed, the choice of k is very critical: a simple initial approach consists in setting $k = \sqrt{n}$ where n is the number of training samples [6].

In step 4, the neighbourhood NN is used by SUNNY algorithm for computing the schedule of solvers $SC = [(s_1, t_1), (s_2, t_2), \ldots, (s_n, t_n)]$ to be executed for solving p. The selected solvers will be executed sequentially in step 5, according to the given time limit. The default timeout T is 1800 seconds (i.e., the one used in the last International CSP Competition) while the default backup solver B is Chuffed. However, both T and B are options that can be specified by the end user. If a solver aborts its execution prematurely (e.g., due to memory overflows or unsupported constraints) its remaining execution time is allocated to the next scheduled solver, if any, or to a not scheduled solver of the portfolio otherwise.

Note that in case p is a CSP the scheduled solvers are executed independently, i.e., there is no cooperation and communication between them.[2] If instead p is a COP instance, `sunny-cp` runs a timesplit solver (see Def. 5.5): it exploits the best solution found by a solver for narrowing the search space of the following ones as seen in Section 5.2. If p is a CSP, `sunny-cp` may output three alternatives: satisfiable (a solution exists for p), unsatisfiable (p has no solutions) or unknown (`sunny-cp` is not able to say anything about p). If p is a COP, there are two more alternatives:

[2] `sunny-cp` views the constituent CSP solvers as "black boxes". The lack of a standard protocol to extract and share knowledge between solvers makes it hard to communicate potentially useful information like nogoods.

sunny-cp can be able to prove the optimality of the solution found or even to prove the unboundedness of p. In particular, the output produced by sunny-cp is conform to the output specification of [22].

According to the methodology and the definitions introduced in Section 5.2, sunny-cp is actually a Timesplit Portfolio Solver (TPS). In particular it allows to solve a given COP by first running a precomputed static schedule in the first $C < T$ seconds. If p is still not solved within such C seconds, the dynamic schedule computed by SUNNY will be executed for the remaining $T - C$ seconds. The static schedule is empty by default since, as we seen in Section 5.2.3.1, its computation may involve a non-trivial off-line phase. Nevertheless, the user has the possibility to set his own static schedule or even to choose among some other static schedules that we have already pre-computed.

sunny-cp is mainly written in Python. It requires the independent installation of the features extractor mzn2feat-1.0 and of each constituent solver. Currently it is not completely portable, partly because some of its constituent solvers (i.e., Chuffed and G12/Gurobi) are not publicly available. The source code used in the MZC 2014 is however free and publicly available at https://github.com/jacopoMauro/sunny-cp/tree/mznc14.

6.1.1 Usage

This section provides a brief overview on the practical use of sunny-cp, taking as reference the version used in MZC 2014. Table 6.1 summarizes the input arguments accepted by sunny-cp. More details are given by typing sunny-cp --help. The user can set from command line the parameters explained above. As an example, in the MZC 2014 we set the option -P for excluding G12/CBC solver from the portfolio (indeed, having a far better MIP solver like G12/Gurobi in the portfolio, the presence of G12/CBC would not have been fruitful). Furthermore, in one of the two version we sent to the MZC we used the -s parameter for defining a static schedule. As expected, this setting turned out to be beneficial.

Option	Description
-h, --help	Prints usage information
-T	Sets the timeout of the underlying SUNNY algorithm
-k	Sets the neighbourhood size of the underlying SUNNY algorithm
-P	Sets the (sub-)portfolio of the underlying SUNNY algorithm
-b	Sets the backup solver of the underlying SUNNY algorithm
-K	Sets the knowledge base of the underlying SUNNY algorithm
-s	Sets a static schedule to be run before SUNNY algorithm
-d, --keep	Utilities for dealing with temporary files

Table 6.1: sunny-cp parameters.

```
minizinc-1.6/examples$ sunny-cp zebra.mzn
% Extracting features...
% Computing solvers schedule...
% Resulting schedule: [('g12cpx', 1005), ('chuffed', 795)]
% Executing solvers...
% Executing g12cpx for 1005 seconds.
animal = array1d(0..4, [4, 1, 2, 5, 3]);
colour = array1d(0..4, [3, 5, 4, 1, 2]);
drink = array1d(0..4, [5, 2, 3, 4, 1]);
nation = array1d(0..4, [3, 4, 2, 1, 5]);
smoke = array1d(0..4, [3, 1, 2, 4, 5]);
----------
% Search completed by g12cpx
```

Figure 6.2: sunny-cp on a zebra puzzle problem.

Let us see now a running example of how sunny-cp works on an instance of the *Zebra Puzzle* CSP taken from the MiniZinc suite. As can be seen from the snapshot in Figure 6.2, sunny-cp can be set up and executed just like a regular CP solver. It first extracts the features, then computes and runs the solvers schedule. In this case, the solver CPX is able to solve almost instantaneously the zebra.mzn problem instance. Note that the output of sunny-cp is by default the output produced by the FlatZinc interpreter of the running constituent solver. If one wish to print a formatted output, it is enough to use the solns2out tool provided by MiniZinc suite.

```
minizinc-1.6/benchmarks/rcpsp$ sunny-cp -s g12cpx,1,minisatid,250 rcpsp.mzn J120_9_6.dzn
% Extracting features...
% Computing solvers schedule...
% Resulting schedule: [('g12cpx', 1), ('minisatid', 250), ('gecode', 740.6350744504846),
('minisatid', 451.8984479634444), ('chuffed', 356.466477586071)]
% Executing solvers...
% Executing g12cpx for 1 seconds.
% o__b__j__v__a__r = 117;
objective = 117;
----------
%  ...Omitting other sub-optimal solutions (objective = 115, 112, ..., 102) found by CPX ...
----------
% o__b__j__v__a__r = 101;
objective = 101;
----------
% o__b__j__v__a__r = 101
% Search not yet completed.
% Adding constraint objective  < 101
% Executing minisatid for 250 seconds.
objective = 100;
% o__b__j__v__a__r = 100;
----------
objective = 99;
% o__b__j__v__a__r = 99;
----------
objective = 98;
% o__b__j__v__a__r = 98;
----------
% Search completed by minisatid
==========
```

Figure 6.3: `sunny-cp` on a RCPSP problem. For the sake of readability, the sub-optimal solutions $115, 112, ..., 102$ found by CPX, as well as other output variables, are here omitted.

We conclude the section by showing how `sunny-cp` behaves on the example depicted in Figure 5.10, where an instance of the RCPSP minimization problem is solved by the timesplit solver $[(\text{CPX}, 1), (\text{MinisatID}, 1799)]$. For running such schedule solver we simply impose a corresponding static schedule via the `-s` option, as shown in Figure 6.3. Note that in this case we set a timeout of 250 seconds for MinisatID: if after that time the instance is still not solved the dynamic schedule

returned by SUNNY is run in the remaining $1800 - (1 + 250) = 1549$ seconds. However, as already shown in the example of Figure 5.10, MinisatID is able to complete the search in less than 250 seconds by exploiting the bound 101 instantaneously found by CPX. Note also that for solving COPs `sunny-cp` introduces an auxiliary variable called `o__b__j__v__a__r` for keeping track of the best value found so far (and, consequently, for possibly injecting that bound to the next scheduled solvers).

6.2 Validation

Extensive evaluations of SUNNY against different state-of-the-art approaches have been reported in Chapters 4 and 5 by using big test sets (i.e., about 500 instances or more) and fairly large solving timeouts (i.e., $T = 1800$ seconds). All these empirical evaluations has proven the effectiveness of SUNNY, in particular w.r.t. the constituent solvers of its portfolio that have been always greatly outperformed. In this section we show instead how `sunny-cp` behaved in the MZC 2014. In this setting things are different: the test set is far smaller (i.e., 100 instances, almost always satisfiable), the timeout is lower (900 seconds), and especially a different evaluation metric is adopted w.r.t. the `proven`, `time`, `score`, and `area` metrics introduced in Def. 4.1, 4.2, 5.1, 5.2, 5.3, 5.6. Indeed, the MiniZinc Challenge Score (MZCS) introduced in Section 5.1.1 and evaluated —by means of simulations— in Section 5.1.3.4 is based on a *Borda count* voting system [42] where each CP problem is treated like a voter who ranks the solvers, and each solver gets a score proportional to the number of solvers it beats.

Two versions of `sunny-cp` attended the competition: *sunny-cp-open* and *sunny-cp-presolve-open*. The first is the default `sunny-cp` solver as described in Section 6.1. The second is instead the variant that runs for every COP a static schedule in the first 30 seconds. In particular the solvers Chuffed, Gecode, and CPX are executed for 10 seconds each. Note that, in contrast to what done in 5.2.3.1, no Set Covering problem was solved for computing the static schedule. The static schedule used in the competition was set "by hand" after some empirical investigations, with

the main goal of quickly solving the easiest instances. *"-open"* refers to *open search category*, i.e., the track in which `sunny-cp` competed.

The results of such track are summarized in Table 6.2. Before discussing the results, it is appropriate to make a few remarks.

SOLVER	SCORE
Chuffed-free*	1324.02
OR-Tools-par (GOLD MEDAL)	*1084.97*
Opturion CPX-free (SILVER MEDAL)	1079.02
sunny-cp-presolve-open	**1064.46**
Choco-par (BRONZE MEDAL)	*1005.61*
iZplus-par	*994.32*
sunny-cp-open	**967.14**
G12/LazyFD-free*	782.78
HaifaCSP-free	779.72
Gecode-par	*720.97*
SICStus Prolog-fd	708.51
Mistral-free	703.56
MinisatID-free*	587.24
Picat SAT-free	586.64
JaCoP-fd	549.24
G12/FD-free*	526.26
Picat CP-free	402.88
Concrete-free	353.24

Table 6.2: MZC 2014 open track. Parallel solvers are in italics, while the solvers included in `sunny-cp` are marked with *.

First, the open class includes also parallel solvers. This can be disadvantageous for sequential solvers like `sunny-cp` that do not exploit more computation units. Second, in the MZC the solving time $\texttt{time}(p, s)$ refers to the time needed by s for solving the FlatZinc model p_s resulting from the conversion of the original MiniZinc

model p to p_s, thus ignoring the solver dependent time needed to flatten p in p_s. The choice of discarding the conversion penalizes a portfolio solver. Indeed, given the heterogeneous nature of our portfolio, `sunny-cp` can not use global redefinitions that are suitable for all its constituent solvers. Therefore, differently from all the other solvers of the open search track, the result of `sunny-cp` were computed by considering not only the solving time of the FlatZinc models but also all the conversion times of the MiniZinc input, including an initial conversion to FlatZinc required for extracting the feature vector of p. Moreover, as we seen in Section 5.1.1, the grading methodology of MZC further penalizes a portfolio solver because in case of ties it may assign a score that is disproportionate w.r.t. the solving time difference. This is a drawback for `sunny-cp` since, even if it selects the best constituent solver, it requires an additional amount of time for extracting the features and for the solver selection. This holds especially in the presence of a clear dominant solver, like Chuffed for the MZC 2014. All these difficulties have been recognized by the organizers, which awarded to `sunny-cp` an *honourable mention*.[3]

Let us now analyse the results. As said, if on the one hand having Chuffed in the portfolio was undoubtedly advantageous for us, on the other hand this can also be counterproductive. Indeed, it is impossible to beat it in the numerous times in which it is the best solver, even if `sunny-cp` selects it to solve the instance. Moreover, note that the other solvers of `sunny-cp` enrolled in the Challenge have not achieved excellent performance: G12/LazyFD is 8^{th}, MinisatID is 13^{th} and G12/FD is 16^{th}. Gecode-par is 10^{th}, but `sunny-cp` did not use this version since Gecode-par is a parallel solver. Finally, G12/CBC, G12/Gurobi, and CPX have not attended the challenge.[4] This setting is inevitably detrimental for a portfolio. There is a clearly dominant constituent solver, while the others have a rather low contribution. To boost the performances of `sunny-cp` we could have used an *'ad hoc'* training set for the MZC. Indeed, as pointed out also in [176], the good results of a portfolio solver

[3] `sunny-cp` was not eligible for prizes, since it contains solvers developed by the MZC organizers.

[4] Note that Gecode-free, the sequential version of Gecode-par, in the open category would be ranked 12^{th}. Opturion CPX[150] is instead based on CPX, but *it is not* CPX. Moreover the constituent solvers of the portfolio may be obsolete w.r.t. the version submitted to MZC 2014.

on a competition can be attributed to the fact that it could be over-trained. We instead preferred to measure sunny-cp performance by using the default knowledge base, because we believe that this setting is more robust, less prone to overfitting, and more suitable for scenarios where Chuffed is not the dominant solver. Indeed, despite the MZC is surely a valuable setting for evaluating CP solvers, it is important that a solver is designed to be good at solving (class of) CP problems rather than being well ranked at the MZC.

Looking at Table 6.2, sunny-cp-presolve-open achieved the 4^{th} position, reaching 97.32 points more than sunny-cp-open (7^{th}). As shown also in Section 5.2, the static schedule resulted in an increase in performance. Here, however, the score difference is mainly due to the higher speed of sunny-cp-presolve-open in case of indistinguishable answer, rather than the higher quality of the solutions. Indeed, in just nine cases sunny-cp-presolve-open gave a better answer than sunny-cp-open, while there are also six cases in which sunny-cp-open is better.

If we just focus on the `proven` and `time` metrics introduced in Def. 4.1, 4.2 for CSPs and 5.1, 5.2 for COPs, we can observe significant variations in the rank. Table 6.3 shows that, by considering `proven`, the two versions of `sunny-cp` would be 2^{nd} and 3^{rd}. Chuffed is still the best solver, but sometimes it is outperformed by `sunny-cp`. For instance, unlike Chuffed, `sunny-cp` is able to solve all the 25 CSPs of the competition. Again, note that `sunny-cp` is not biased towards Chuffed. Figure 6.4 shows the number of times (in percentage) a constituent solver of `sunny-cp` completes the search. As can be seen, almost all the solvers find an optimal solution at least one time. The contribution of Chuffed is not massive: about one third of the instances. Looking at the average `time` results of Table 6.3, we notice that `sunny-cp` is overtaken by Opturion. This is not surprising: `sunny-cp` proves more optima, but Opturion is on average faster.

Looking at the results on the individual instances, on nine problems `sunny-cp` is able to *outperform* all its constituent solvers participating in the MZC. For four instances it is gold medallist, and for the *cyclic-rcpsp* problem (five instances) it is the overall best solver of the competition.

SOLVER	#proven	SOLVER	time
Chuffed-free	68	Chuffed-free	331.27
sunny-cp-presolve-open	**59**	Opturion CPX-free	461.05
sunny-cp-open	**57**	**sunny-cp-presolve-open**	**469.86**
Opturion CPX-free	53	**sunny-cp-open**	**488.58**
OR-Tools-par	50	OR-Tools-par	491.29
G12/LazyFD-free	46	G12/LazyFD-free	511.28
MinisatID-free	46	Choco-par	553.83
Picat SAT-free	44	Gecode-par	563.65
Choco-par	43	MinisatID-free	566.24
SICStus Prolog-fd	41	iZplus-par	570.30
iZplus-par	41	Picat SAT-free	570.31
Gecode-par	40	SICStus Prolog-fd	598.65
HaifaCSP-free	39	HaifaCSP-free	599.85
Mistral-free	37	Mistral-free	606.75
JaCoP-fd	31	JaCoP-fd	659.17
G12/FD-free	27	G12/FD-free	693.99
Concrete-free	22	Concrete-free	737.02
Picat CP-free	17	Picat CP-free	777.84

Table 6.3: MZC results considering `proven` and `time` metrics.

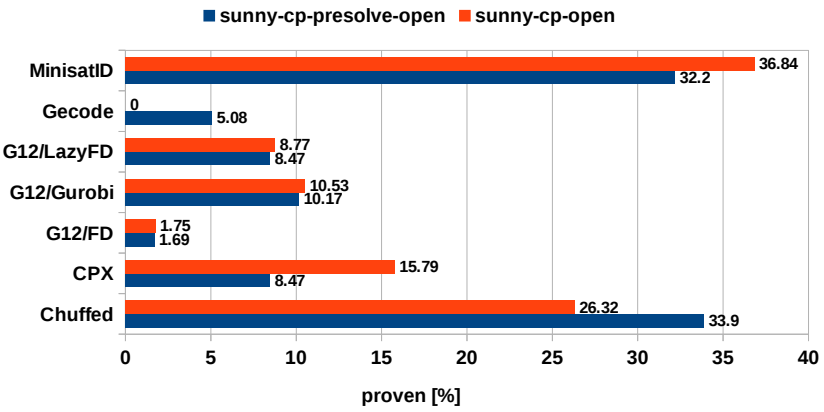


Figure 6.4: Solvers contribution in terms of `proven`.

We conclude the section by proposing an alternative ranking system. As previously mentioned, we realized that, in case of indistinguishable answer between two solvers, in the MZC the gain of a solver depends on the runtime ratio rather than the runtime difference. For instance, let us suppose that two solvers s_0 and s_1 solve a problem p in 1 and 2 second respectively. The score assigned to s_0 is $2/3 = 0.667$ while s_1 scores $1/3 = 0.333$. The same score would be reached by the two solvers if $\texttt{time}(p, s_1) = 2 * \texttt{time}(p, s_0)$. Hence, if for example $\texttt{time}(p, s_0) = 400$ and $\texttt{time}(p, s_1) = 800$, the difference between the scores of s_0 and s_1 would be the same even if the absolute time difference is 1 second in the first case, 400 seconds in the second. In our opinion, this scoring methodology could overestimate small time differences in case of easy instances, as well as underrate big time differences in case of medium and hard instances. A possible workaround to this problem is to assign each solver a score in $[0, 1]$ that, in case of indistinguishable answer between two solvers, is linearly proportional to the solving time difference. We then propose a modified metric `MZC-MOD` that differs from the MZC score since, when the answers of s_0 and s_1 are indistinguishable, it gives to the solvers a "reward" such that:

$$\texttt{MZC-MOD}(p, s_i) = 0.5 + \frac{\texttt{time}(p, s_{1-i}) - \texttt{time}(p, s_i)}{2\,T} \qquad i \in \{0, 1\}$$

where T is the timeout (i.e., 900 seconds for MZC).

With this new metric, according to the previous examples, if $\texttt{time}(p, s_0) = 1$ and $\texttt{time}(p, s_1) = 2$, then the score difference is minimal since $\texttt{MZC-MOD}(p, s_0) = 0.5 + 1/2 * 900 = 0.501$ and $\texttt{MZC-MOD}(p, s_1) = 0.5 - 1/2 * 900 = 0.499$. In contrast, if $\texttt{time}(p, s_0) = 400$ and $\texttt{time}(p, s_1) = 800$ then the difference is proportionally higher: $\texttt{MZC-MOD}(p, s_0) = 0.722$ and $\texttt{MZC-MOD}(p, s_1) = 0.278$.

Figure 6.5 depicts the effects of using `MZC-MOD` in place of MZC score w.r.t. the first seven positions of the ranking. The classification with the new score is much more compact and reflects the lower impact of the solving time difference in case of identical answers. Chuffed is firmly in the lead, but if compared to the original score loses about 165 points. Except for `sunny-cp` and Choco, all other approaches have a deterioration of performance. In particular, sunny-cp-presolve-open gains 19.82

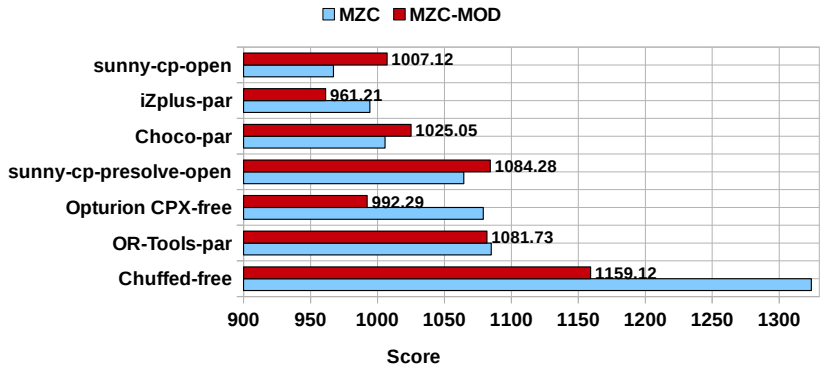


Figure 6.5: Score differences between MZC score and `MZC-MOD`.

points surpassing OR-Tools-par in the 2nd position. Even more noticeable is the score improvement of sunny-cp-open, that earns about 40 points and 2 positions in the ranking.

6.3 Summary

In this chapter we described `sunny-cp`, a sequential portfolio solver able to solve both satisfaction and optimization problems defined in MiniZinc language. `sunny-cp` is aimed to provide a flexible, configurable, and usable CP portfolio solver that can be set up and executed by the end users just like an usual, single CP solver.

Since we already verified in former evaluations the effectiveness of `sunny-cp` when validated on heterogeneous and large sets of test instances, in this section we focused on the results obtained by two versions of `sunny-cp` in the MiniZinc Challenge 2014. Despite the grading mechanism of the challenge may penalize a sequential portfolio solver, `sunny-cp` turned out to be competitive even when compared to parallel solvers, and it was sometimes even able to outperform state-of-the-art constraint solvers. In this regard, we also proposed and evaluated an alternative scoring system that, in case of indistinguishable quality of the solutions, takes into account the solving time difference in a more linear way.

Our claim about `sunny-cp` is not related to the introduction of a new approach, but concerns the presentation of a new tool that could serve as a baseline for future

developments. We hope that sunny-cp can take a step forward in encouraging and disseminating the actual use of CP portfolio solvers.

There is plenty of lines of research that can be explored in order to extend the capabilities of sunny-cp. For examples, the selection strategies of [114, 151, 132], the impact of using different distance metrics and features [124], the robustness towards the neighbourhood size of k-NN algorithm [10], the automated tuning of different solvers configurations [114], the use of benchmark generation techniques [102], the evaluation of different ranking methods [26, 75]. Certainly one of the most promising directions for further research is the extension of sunny-cp to a parallel environment, where multiple cores can be used to launch more than one constituent solver in parallel. In this setting it may be important to devise and evaluate cooperative strategies between the constituent solvers, by exploiting and sharing tokens of knowledge like the nogoods generated by the lazy clause generation solvers.

Chapter 7

Conclusions and Extensions

"Ite, missa est"[1]

"Multae Manus Onus Levant" is a Latin motto that literally means "Many hands lighten the load". If we see the hands like different constraint solvers, and the load like an hard combinatorial problem to be solved, we can get an idea of the underlying logic of portfolio solvers. In this thesis we investigated the benefits of portfolio solving in the CP context, trying to improve a state-of-the-art that in such a setting still turns out to be rather immature and unexplored. We have embarked on a path of investigations, observations, insights, implementations, and extensive evaluations. We decided to first start with CSPs, which are closer to SAT problems (that can be in turn be seen as particular cases of CSPs). Then, we moved to the less explored field of portfolio approaches for solving COPs. Being a COP more general than a CSP, here things become more complicated and, at the same time, more challenging. The promising results observed in these experimental evaluations led us to merge these two lines of research for building `sunny-cp`, a prototype for a new generation CP portfolio solver. More in detail, the original contributions of this thesis are the following:

[1] Concluding Latin words addressed to the people in the Mass of the Roman Rite, as well as the Lutheran Divine Service.

- we performed several empirical evaluations of different portfolio approaches for solving both CSPs and COPs. We think that these comparative studies, that required a considerable effort, can be an useful baseline to compare and develop new portfolio approaches;

- we provided the full support of MiniZinc, nowadays a de-facto standard for modelling and solving CP problems, while still retaining the compatibility with XCSP format. In particular, we developed tools for converting a XCSP instance to MiniZinc (i.e., `xcsp2mzn`) and for retrieving an extensive set of features from a MiniZinc model (i.e., `mzn2feat`);

- we developed SUNNY, a lazy portfolio approach for constraint solving. SUNNY is a simple and flexible algorithm portfolio that, without building an explicit prediction model, predicts a schedule of the constituent solvers for solving a given problem. The good performances reached by SUNNY on CSPs led us to extend it to COPs and to build some prototypes of CSP/COP portfolio solvers (i.e., `sunny-csp`, `sunny-ori`, `sunny-com`, and `sunny-tps`);

- we took a step forward towards the definition of COP portfolios. We introduced new metrics like `score` and `area` and we studied how CSP portfolio solvers might be adapted to deal with COPs. In particular, since COP solvers may yield sub-optimal solutions before finding the best one, we addressed the problem of boosting optimization by exploiting the sequential cooperation of different COP solvers. Empirical results showed that this approach may even allow to outperform the best solver of the portfolio;

- we finally reaped the benefits of our work for developing `sunny-cp`: a new tool aimed at solving a generic CP problem, regardless of whether it is a CSP or a COP. The aim of `sunny-cp` is to provide a flexible, configurable, and usable CP portfolio solver that can be set up and executed just like a regular individual CP solver. To the best of our knowledge, `sunny-cp` is currently the only sequential portfolio solver able to solve generic CP problems.

We think that this work could serve as a cornerstone for the effective use and dissemination of portfolios in (and also outside) the CP field. Indeed, despite the proven effectiveness showed in several experiments, the actual use of portfolio solvers is pretty limited and usually confined to annual competitions. This is obviously restrictive: if for winning a competition based on relatively few instances a portfolio solver requires a year of training, it is evident that its usability in other settings is practically zero. There is plenty of future directions. For instance, you might consider well-known problems like feature selection, dataset filtering, or algorithm configuration. Other interesting investigations could be the impact assessment of different performance metrics, or the inclusion in the portfolio of incomplete, non-deterministic solvers like local search solvers.

We identified in particular four main challenges for the future of CP portfolio solvers:

- *prediction model*: often the prediction model responsible for the solver selection requires an heavy training phase. We think that one of the main future directions for portfolio solvers is the reduction of the training costs. For instance, the SUNNY approach can not be defined purely dynamic: it still needs a detailed knowledge base for making predictions. Making the portfolio solvers more flexible and dynamic could certainly be useful in the future.

- *optimization problem*: since the state of the art for COP portfolios is still in an embryonic stage, we think that a deepening on optimization problems is needed. We suggest to insist on collaborative portfolios able to exchange information between the constituent solvers. Given the benefits of bounds communication, it would be interesting to share also other information (e.g., redundant constraints like nogoods).

- *parallelisation*: working with different solvers in parallel is less trivial than thought at first, and it is not only a matter of software engineering. Synchronization and memory consumption issues arise, and the communication between constituent solvers is not so straight as for sequential portfolios. This

direction could open interesting streams of research, especially when applied to optimization problems.

- *utilization*: facilitating the practical use of portfolio solvers for solving generic CP problems is desirable. As an example, we realized that is often very difficult to use the original portfolio solver for making experiments. It would certainly be a good practice in the future to the develop free and usable code and APIs for encouraging the spread of CP portfolio solvers, as well as having standard benchmarks and a standard language for encoding CP problems.

We are currently addressing these challenges by developing a more configurable and usable version of `sunny-cp`, also capable of exploiting the simultaneous and cooperative execution of its constituent solvers in a multicore setting.

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